Liuc Papers

Pubblicazione periodica dell'Università Carlo Cattaneo - LIUC

Numero 296, giugno 2016

Serie

Metodi quantitativi 23

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Serie: Metodi Quantitativi

Liuc Papers

ISSN:1722-4667

Direttore Responsabile: Piero Cavaleri
Direzione, redazione, amministrazione: Università Carlo Cattaneo - LIUC
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Registro stampa Tribunale di Busto Arsizio n. 11/93 del 11.06.93

Comunicazioni di carattere organizzativo vanno indirizzate a:
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A NONLINEAR POLICY FOR TRADING IN INDEX FUNDS

Luca Ghezzi[1], *Fernanda Strozzi*[2]

1. Introduction

Neoclassical finance rests on the **efficient market hypothesis**, whereby stock prices immediately and accurately incorporate any new piece of relevant information owing to the trading of the most skilful operators. As the relevant information comes up at random, the changes in stock prices, i.e. the rates of return, are almost unpredictable. Moreover, as no systematic errors are usually made by the above-mentioned operators, both statistical and financial analyses can't end up in excess returns. As consequence, no stock index can be outperformed consistently by a trading or investment policy that has a higher mean return as well as the same standard deviation (risk) of the stock index or a lower one.

Some empirical evidence about the efficient market hypothesis is outlined in Section 2 along with its neoclassical and behavioural interpretations. According to Farrell (1997, chapt. 1), a portfolio manager and lecturer,

- stock markets are not entirely efficient, with developed stock markets tending to be more efficient than emerging markets, because they are more extensively analysed;
- some inefficiencies tend to be removed owing to both active learning and competitive imitation.

Needless to say, those conditions and developments would be precluded from the lack of financial analysis, trading, and active management; this is an important motivation for further applied research. As explained by Tobin (1984), the above-mentioned **informational** efficiency of stock markets goes along with the **functional** efficiency of a financial system, which is important to households, firms, and governments.

This paper expands on Strozzi and Zaldìvar (2005) as well as Strozzi and Zaldìvar Comenges (2006), who focus on trading in foreign currencies; they develop a family of nonlinear trading policies and test it successfully on high-frequency data about the exchange

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rates between the US dollar and 18 other currencies. As shown by Strozzi *et al.* (2002), the underlying and unknown nonlinear dynamical systems are supposed to be in a **transient** phase; therefore, use is made of numerical estimates of their **physical divergences**. As explained by Strozzi and Zaldìvar (2005) as well as Strozzi and Zaldìvar Comenges (2006), this approach has previously been taken in analyzing the **transient** phases of chemical processes, aiming at the early detection of runaway reactions.

In this paper, the above-mentioned nonlinear policies are tested on stocks rather foreign currencies, more precisely on daily data about the Standard & Poor's **S&P 500** stock index. Daily data are complemented by other new features. The data sample spans 15 years rather than one year. The benchmark is now a passive portfolio, i.e. the usual choice both in financial economics and business practice. As a consequence, performance is measured in terms of a reward to risk ratio, i.e. another usual choice in both financial economics and business practice. Moreover, the robustness of our numerical results is carefully checked, as motivated at the end of Section 3.3. More precisely, as a specific nonlinear policy is characterised by a triplet of parameters, some triplets are shown to be either **satisfactory** or **unsatisfactory** in four data samples, including the first 5, 10, or 15 years as well the years 2006-2010. Indeed, satisfactory triplets, i.e. satisfactory nonlinear trading policies, outperform the benchmark resulting in a higher reward (daily mean return and hence final accumulation) and a lower risk (daily standard deviation).

The paper is organised as follows. Some empirical evidence in favour of or against the efficient market hypothesis is briefly reviewed in Section 2. The data set is presented in Section 3 along with the family of nonlinear trading policies; peculiar attention is devoted to the measurement of their performance. The analysis of our numerical results is carried out in Section 4, where the nonlinear trading policies are tested against the **S&P 500** stock index, their forecasting power being contrasted with that of the Dow Theory, as reported by Brown *et al.* (1998). Moreover, the nonlinear trading policies are also tested against the null hypothesis that the S&P 500 stock index follows a random walk. Finally, two promising extensions are highlighted. Conclusions are given in Section 5.

2. The efficient market hypothesis

There is a whopping body of **empirical evidence** about the efficient market hypothesis, an educated summary being provided by the textbook Elton *et al.* (2010, chapt. 17). Although many empirical papers are in favour of the hypothesis, several so called **anomalies** are documented, including: an inertia of equity portfolios, a dissimilar behaviour of growth and value stocks, a long-term mean reversion in stock indices, the tendency displayed by the very

best and worst equity mutual funds to repeat their recent performance. Such anomalies add to the empirical properties listed in the review Chakraborti *et al.* (2011) of empirical studies in Econophysics.

Nonetheless, most actively managed **mutual funds**, be they bond or equity funds, have underperformed their benchmarks (Elton *et al.*, 2010, chapt. 25). However, although the managers of mutual funds as a whole haven't display enough skill to cover their expenses, the managers of **hedge funds** have to some extent: as remarked by (Jackson, 2003, chapt. 9), the CSFB Tremont index outperformed the S&P 500 stock index over the years 1994-2002, its mean return being higher and its standard deviation (risk) lower. Hedge funds are more opaque and less regulated than mutual funds, their total assets under management being much lower. Notably, the management style of a hedge fund can be **discretionary**, **systematic**, or in between. In other words, it can be based on subjective judgement or computer programs (e.g. aiming at **statistical arbitrage**).

Moreover, the 2013 Nobel Prize in Economic Sciences was also awarded to Eugene F. Fama, a neoclassical economist, and Robert J. Shiller, a behavioural economist. As mentioned by the Royal Swedish Academy of Sciences (2013), their empirical researchs have shown that stock prices, though unpredictable over the short term, i.e. the next year, are predictable over the medium term, i.e. the next 3-5 years, owing to both rational and emotional causes. The more sophisticated study Cochrane (2008) has confirmed such claims but it has also cast doubts on their operational relevance. The vector autoregression in Cochrane (2008) is consistent with a general dividend discount model but it rests on the specific and questionable assumption that dividend yields are the only exogeneous variable that affects both future dividends and stock returns.

Shiller (2003) comments on the empirical evidence whereby **stock indices** may remain above or below their **fundamental values** for several years in a row, fundamental values reflecting the expectations of subsequent dividends, which are hard to estimate. In his opinion, the collective behaviour of traders and investors is both rational and emotional, being driven by news and affected by press coverage. Therefore, stock markets are not **fundamentally** efficient, as also maintained by Tobin (1984); in principle, this doesn't prevent them from being **informationally** efficient. Shiller (2003) lists some instances of emotional decisions; moreover, he distinguishes between more emotional operators and more rational ones, pointing out some reasons why the latter, i.e. the most skilful ones, may be unable to counter the former, when they give wrong buy or sell orders. Such inability clashes with the statement of the efficient market hypothesis.

Such a **behavourial** interpretation of the empirical evidence complements a **neoclassical** interpretation, which claims that stock markets tend to be **fundamentally** efficient as well, as the collective risk tolerance varies with time. More precisely, it is lower (higher) under more risky (less risky) circumstances, e.g. in a recession (expansion), thus driving up (down) dividend yields, i.e. dividend-price ratios.

According to Shiller (2003, p. 96), Haugen (1997, chapt. 6 and chapt. 24), and references therein

- portfolios of winning (losing) stocks display the above-mentioned **inertia** patterns, tending to repeat their performance over the prior 6 to 12 months in the subsequent 6 months;
- on appreciating (depreciating), those portfolios tend to reverse their performance over the subsequent 30 months or so.

Two non mutually exclusive explanations can be given, a **behavioural** and a **neoclassical** one, whereby

- a slow and gradual reaction to favourable (unfavourable) quarterly reports on earnings and dividends is followed by an overreaction to a chain of good (bad) news, caused by more emotional operators. Only later, more rational operators drive back prices toward their fundamental values;
- chains of good (bad) news signal that winning (losing) stocks have become less (more) risky with their prices adjusting accordingly.

According to **technical analysis**, although stock prices reflect all available relevant information, they take time to do so; as the collective beaviour of traders and investors tends to repeat itself over time, price patterns are recognizable and predictable to an extent that depends obviously on experience and skill (Stevens, 2002, chapt. 1; Reuters Ltd., 1999, sect. 1). Therefore, **inertia** patterns are considered by **technical analysis** too. Some empirical studies show that different **technical** trading rules would have outperformed specific stock indices over specific historical periods (Rosillo *et al.*, 2013, sect. 2). One of those **technical** trading rules is based on the indicators **RSI** (relative strength index) and **MACD** (moving average convergence-divergence). When dealing with trading in foreign currencies, Strozzi and Zaldìvar Comenges (2006) compare their divergence-based procedure with the **RSI**. As documented by Rosillo *et al.* (2013, sect. 2), the use of technical analysis is widespread among foreign currency traders.

In the light of this, the forecasting power of the divergence-based procedure is compared with that of the Dow Theory, the seminal form of **technical analysis**, at the end of Section 4.1.

3. Problem statement

3.1 Data set

The Standard & Poor's **S&P 500** is a value-weighted stock index. It is based on a basket of 500 large companies, all listed on the NYSE or NASDAQ. It was devised in 1957 from a stock index based on 233 companies, in existence since 1923. Its numerator is the sum of 500 terms; nowadays, each term is equal to a stock price times the number of free floating stocks. Its denominator is occasionally adjusted, e.g. when a company is replaced by another one. The net total return version takes into account the reinvestment of net dividends in the corresponding stocks.

Our analysis rests on the net total return version; daily data from 12/31/1998 to 12/31/2013 were downloaded from **Bloomberg Finance LP** at the beginning of 2014.

3.2 A family of nonlinear trading policies

Let time t be measured in **days**, SP be the value of the **S&P 500** stock index (net total return version). Consider an **index fund** or an **exchange-traded fund** that replicates the S&P 500 stock index; for simplicity's sake, suppose that SP is also the price of one share of the index mutual fund. Let PV be our accumulation, i.e. the value of our portfolio, which is made up of either n shares of the index fund or cash; suppose that switches from shares to cash or vice versa may occur every δ days.

Let r be the rate of return on our portfolio over a period of δ **days**, i.e.

$$1 + r_{t+\delta} = \frac{PV_{t+\delta}}{PV_t}$$

so that $\log(1 + r_{t+\delta})$ is the rate of **logarithmic** return over a period of δ days. We have

$$PV_{t+\delta} = e^{\log(1+r_{t+\delta})} PV_t \quad \text{with} \quad t = k\delta \quad \text{and} \quad k = 0,1,2,\cdots$$

so that

$$PV_{n\delta} = e^{\log(1+r_{\delta}) + \log(1+r_{2\delta}) + \cdots + \log(1+r_{n\delta})} PV_0$$

Suppose that the latest switch occurred at time τ; we have

$$PV_{t+\delta} = n_\tau SP_{t+\delta} \quad \text{and} \quad 1 + r_{t+\delta} = \frac{PV_{t+\delta}}{PV_t} = \frac{n_\tau SP_{t+\delta}}{n_\tau SP_t} = \frac{SP_{t+\delta}}{SP_t}$$

for a **long** position, i.e. an investment in stocks, $n_\tau = \frac{PV_\tau}{SP_\tau}$ being the number of shares bought at time τ by spending all the money available. We also have

$$PV_{t+\delta} = PV_\tau \quad \text{and} \quad 1 + r_{t+\delta} = \frac{PV_{t+\delta}}{PV_t} = 1$$

for a **riskless** but steady position, PV_τ being cash, available since time τ. Such an assumption is conservative, as cash usually earns interest.

Our family of nonlinear **trading policies** is based on the sign of the second difference $\Delta^2 V_t$, where volume V_t is given by

$$V_t = \det \begin{bmatrix} \ln SP_t - \ln SP_{t-d} & 0 & \cdots & 0 \\ 0 & \ln SP_{t-d} - \ln SP_{t-2d} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \ln SP_{t-(e-1)d} - \ln SP_{t-ed} \end{bmatrix}$$

with d denoting the **time delay** and e the **embedding dimension**. As shown in Kantz and Schreiber (1997), such **reconstruction parameters** allow one to reconstruct the properties of the underlying dynamical system by using only one measurable variable, i.e. a scalar time series. The meaning of volume V_t as well as the link between volume and physical divergence are explained in Strozzi and Zaldìvar (2005, sect. 2.2.1). More precisely, we have

$$n_t = \begin{cases} \frac{PV_t}{SP_t} & \text{for} \quad \Delta^2 V_t < 0 \\ 0 & \text{for} \quad \Delta^2 V_t \geq 0 \end{cases} \tag{1}$$

so that either n_t shares of the index fund are held for $n_t > 0$ or cash is idle for $n_t = 0$; recall that switches may occur every δ days. Some insight into (1) is given by Strozzi and Zaldìvar (2005, sect. 2.4). According to a mechanical analogy, the divergence-based procedure focuses on **acceleration** rather than on **speed** in contrast with other **technical** indicators: a risky position

is taken if acceleration has decreased, whereas a riskless position is taken if acceleration has increased.

All parameters of each triplet (δ, d, e) are positive integers with

$$e = 1 \quad \text{and} \quad d < \delta$$

the educated choice $e = 1$ being in line with some empirical findings (Strozzi *et al.*, 2002). Each triplet (δ, d, e) is matched by only one **trading policy**, which results from the following computations

	V_δ	$V_{2\delta}$	$V_{3\delta}$	$V_{4\delta}$
0	δ	2δ	3δ	4δ

1) $V_\delta = \ln\left(\frac{SP_\delta}{SP_{\delta-d}}\right)$ is computed at time δ;

2) $V_{2\delta} = \ln\left(\frac{SP_{2\delta}}{SP_{2\delta-d}}\right)$ and $\Delta V_{2\delta}$ are computed at time 2δ;

3) $V_{3\delta}$, $\Delta V_{3\delta}$, and $\Delta^2 V_{3\delta}$ are computed at time 3δ; trading starts, the index fund being preferred to cash for $n_{3\delta} > 0$, otherwise cash is preferred to the index fund;

4) $V_{4\delta}$, $\Delta V_{4\delta}$, and $\Delta^2 V_{4\delta}$ are computed at time 4δ, when the forecast made at time 3δ proves either right or wrong. Trading may go on, the index fund being preferred to cash for $n_{4\delta} > 0$, otherwise cash is preferred to the index fund; and so on.

3.3 Performance measurement

Consider a specific trading policy (1), i.e. a specific triplet (δ, d, e); let $\bar{r}$ be the sample average of its logarithmic return and σ be the sample standard deviation. Both sample moments are **annualised**; as 252 is the average number of business days per year in our data sample, $\bar{r}$ is equal to 252 times the sample average per day, whereas σ is equal to $\sqrt{252}$ times the sample standard deviation per day.

The **benchmark** is a **passive portfolio**, i.e. an usual reference in both financial economics (Brown *et al.*, 1998) and business practice (Gibson, 2008, chapt. 13). More precisely, it is an

index fund or **exchange-traded fund** that replicates the **S&P 500** stock index; recall that net dividends are reinvested in the corresponding stocks. Let $\bar{r}_{SP}$ be the sample average of its logarithmic return and σ_{SP} be the sample standard deviation; needless to say, both sample moments are annualised. As explained in Section 3.2, the first forecast can be made only after 3δ days; in other words, the start date depends on δ so that both $\bar{r}_{SP}$ and σ_{SP} depend on δ too.

A specific trading policy (1), its benchmark, and their performance can be compared in terms of their **reward to risk** ratios

$$RR = \frac{\text{annualised mean return}}{\text{annualised standard deviation}} \tag{2}$$

The **reward to risk** ratio (2) is close in spirit to the Sharpe ratio, another usual reference for financial economists as well as financial advisors, financial analysts, and portfolio managers (Farrell, 1997, chapt. 15). It is readily realised that whenever $\bar{r} > \bar{r}_{SP}$, the final portfolio value of a specific trading policy (1) exceeds that of the benchmark. However, as investors are **risk-averse**, a large portfolio value after 15 years is unpalatable, if it is too risky. Therefore, a specific trading policy (1) will outperform the benchmark, only if both $\bar{r}$ and σ are appropriate. Indeed, the reward to risk ratio of a **satisfactory** trading policy (1) is appropriately larger than that of its benchmark, whereas the reward to risk ratio of an **unsatisfactory** trading policy (1) is appropriately smaller than that of its benchmark. As each trading policy (1) also involves a riskless position, we have $\sigma < \sigma_{SP}$ by construction. In general, we don't check whether $\bar{r} > \bar{r}_{SP}$ and $\sigma < \sigma_{SP}$, which is the favourite case.

Needless to say, we can claim that the divergence-based procedure has some **forecasting power** only if it proves **robust**. Accordingly, we will divide our **historical period** into a few subperiods and check whether there exist specific nonlinear trading policies (1), i.e. specific triplets (δ, d, e), that are **satisfactory**, or **unsatisfactory**, in all subperiods. The resort to real-time forecasting, outlined in Section 4.3, should dispel doubts more effectively.

Although our modelling is parsimonious, data mining (White, 2000) is a serious risk. Therefore, we will perform Montecarlo tests (White, 2000) in Section 4.2. More precisely, the forecasting power of the divergence-procedure will be tested against the null hypothesis that the S&P 500 stock index follows a random walk, namely that its logarithmic returns are a white noise.

Keep in mind that such a null hypothesis is just a usual and convenient term of comparison. Actually, the logarithmic returns on the S&P 500 stock index are not independent and identically distributed, the properties of their distribution changing with time (Chakraborti *et al.*, 2011). Therefore, the mean and standard deviation of their distribution are not constant with time; moreover, as absolute or squared returns are autocorrelated, large (small) returns tend to be followed by large (small) returns, of either sign, a phenomenon known as **volatility clustering** (Franses and van Dijk, 2000, chapt. 1). A comparison between the sample moments of Table 1 and Table 4 is rough but insightful; indeed, data samples should begin and end with similar financial multiples (Bernstein, 1997), e.g. similar price-earnings ratios.

4. Numerical results

In the sequel, the nonlinear trading policies (1) are tested against the **S&P 500** stock index. Both their performance and robustness are carefully checked. All the numerical results have been obtained in MATLAB.

4.1 Performance and robustness in the case of the S&P 500 stock index

In Figure 1, the trading policies (1) and the passive policies are compared by plotting their **reward to risk** ratios *RR* as a function of the parameters δ and *d*, which are respectively lower than or equal to 30 and 25 business days. More precisely, Figure 1a is based on the first 5 years, Figure 1b is based on the first 10 years, Figure 1c is based on the years 1999-2013, i.e. the whole data sample, and Figure 1d is based on the years 2006-2010; keep in mind that the outbreak of the 2nd global financial crisis occurred in 2008. All the above-mentioned four contour plots display a striking similarity, i.e. they display white areas where the corresponding trading policies (1) outperform the passive policies in terms of **reward to risk** ratio. For instance, one white area is smaller and approximately such that $10 \le \delta \le 11$ and $5 \le d \le 6$, whereas another white area is larger and such that $16 \le \delta \le 19$ and $10 \le d \le 13$.

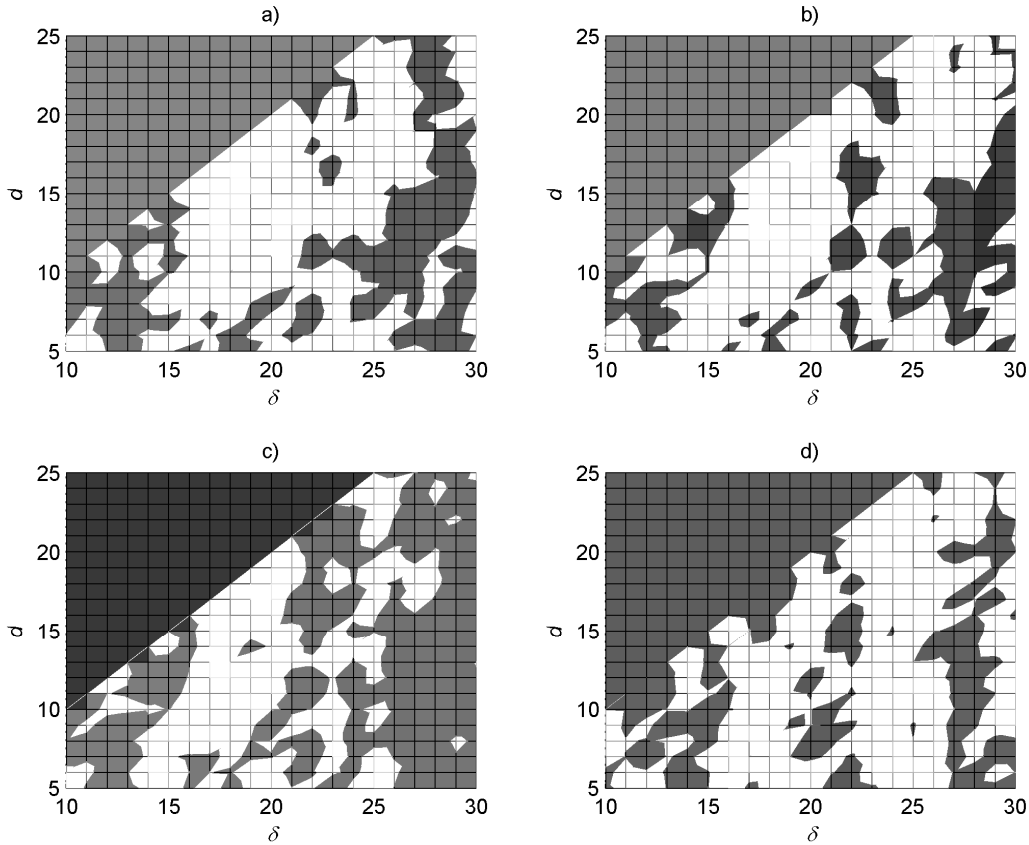

Figure 1 – Comparison of reward to risk ratios $RR_{trading}$ and $RR_{passive}$ in four different historical periods (a: 1999-2003; b: 1999-2008; c: 1999-2013; d: 2006-2010). We have $RR_{trading} > RR_{passive}$ in the white regions of the four contour plots and $RR_{trading} < RR_{passive}$ in the grey regions. Moreover, the nonlinear trading policy (1) is unfeasible in the upper-left triangle, which is dark and such that $\delta < d$.

In Figure 2 four instances of trading policies (1) are considered. We have $\delta = 11$ and $d = 6$ in Figure 2a, $\delta = 18$ and $d = 12$ in Figure 2b, $\delta = 12$ and $d = 7$ in Figure 2c, $\delta = 27$ and $d = 10$ in Figure 2d. All the above-mentioned four diagrams compare a specific trading policy (1) with the corresponding passive one by portraying the time patterns of their portfolio values over the years 1999-2013, i.e the whole data sample. Notice that the switches of the nonlinear trading policies are not very frequent, which curbs commissions and fees due to trading. Figure 2a and 2b display two instances of very **satisfactory** trading policies (1), whereas Figure 2c and 2d display two instances of very **unsatisfactory** trading policies (1). Remarkably, the final portfolio values of the two satisfactory trading policies are definitely higher than those of the two corresponding passive policies.

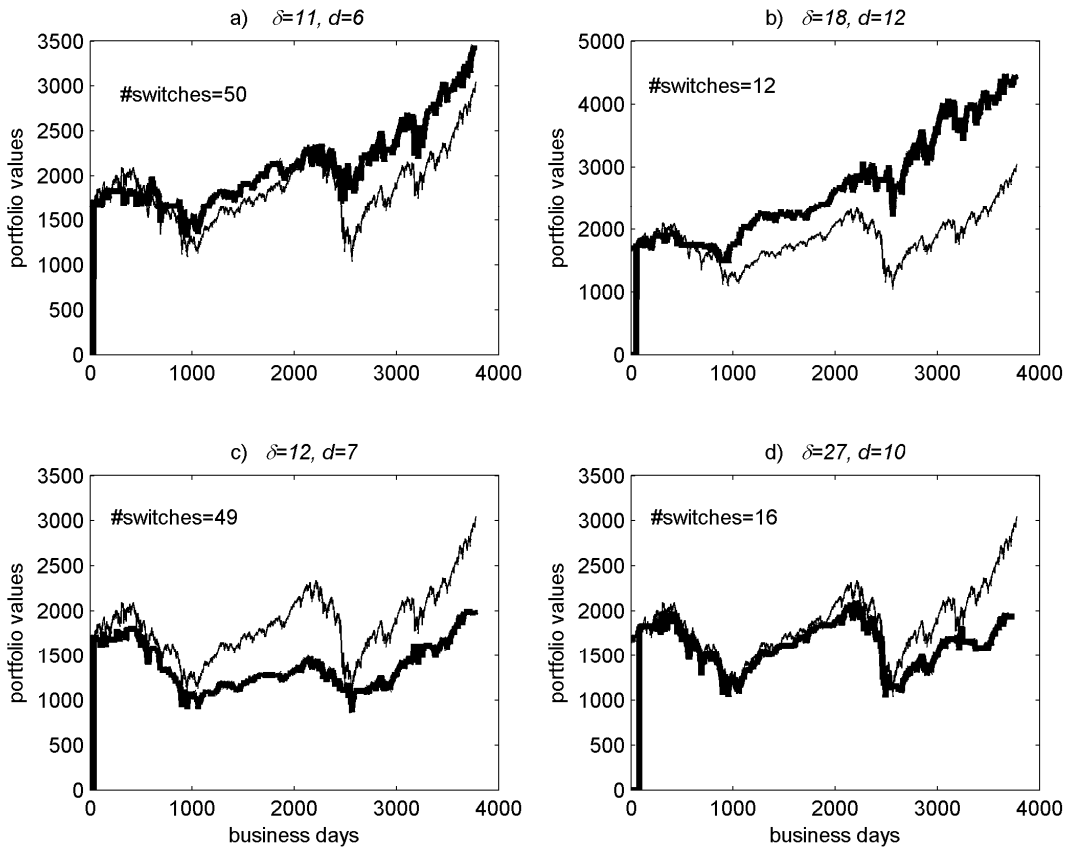

Figure 2 – Nonlinear trading policies (bold lines) versus passive policies. Time patterns of their portfolio values for the years 1999-2013 and four different pairs of parameters δ and d.

As reported in Table 1, both **satisfactory** trading policies are such that $\bar{r} > \bar{r}_{SP}$ and $\sigma < \sigma_{SP}$, whereas both **unsatisfactory** trading policies are such that $\bar{r} < \bar{r}_{SP}$ and again $\sigma < \sigma_{SP}$.

Table 1 – Nonlinear trading policies versus passive policies. Reward to risk ratios $RR_{trading}$ and $RR_{passive}$ for the years 1999-2013 and four different pairs of parameters δ and *d*.

δ	*d*	$\bar{r}$	σ	$RR_{trading}$	$\bar{r}_{SP}$	σ_{SP}	$RR_{passive}$
11	6	4,84%	17,68%	0,27	3,85%	20,68%	0,19
18	12	6,30%	15,05%	0,42	3,55%	20,70%	0,17
12	7	0,99%	16,79%	0,06	3,67%	20,66%	0,18
27	10	0,39%	18,90%	0,02	3,32%	20,73%	0,16

As reported in Table 2, all three subperiods are similar to the period 1999-2013 of Table 1, which is a clue to robustness. Indeed, both **satisfactory** trading policies are such that $\bar{r} > \bar{r}_{SP}$ and $\sigma < \sigma_{SP}$. Although $\bar{r}_{SP} < 0$ in two subperiods, both **satisfactory** policies are such that $\bar{r} > 0$.

Table 2 – Nonlinear trading policies versus passive policies. Reward to risk ratios $RR_{trading}$ and $RR_{passive}$ in three different subperiods (1999-2003; 1999-2008; 2006-2010) and for four different pairs of parameters δ and *d*.

δ	*d*	years	$\bar{r}$	σ	$RR_{trading}$	$\bar{r}_{SP}$	σ_{SP}	$RR_{passive}$
11	6	99-03	1,54%	17,78%	0,09	-2,22%	21,20%	-0,10
		99-08	1,68%	18,06%	0,09	-2,45%	21,15%	-0,12
		06-10	4,77%	18,34%	0,26	0,54%	25,41%	0,02
18	12	99-03	5,16%	15,15%	0,34	-3,09%	21,27%	-0,15
		99-08	4,86%	13,71%	0,35	-2,87%	21,13%	-0,14
		06-10	10,26%	18,96%	0,54	0,10%	25,62%	0,004
12	7	99-03	-8,78%	17,92%	-0,49	-2,21%	21,15%	-0,10
		99-08	-4,35%	15,73%	-0,28	-2,57%	21,15%	-0,12
		06-10	-8,17%	23,18%	-0,35	0,79%	25,39%	0,03
27	10	99-03	-5,02%	20,41%	-0,25	-3,93%	21,29%	-0,18
		99-08	-3,89%	19,39%	-0,20	-2,81%	20,17%	-0,14
		06-10	0,46%	22,88%	0,02	0,12%	25,88%	0,005

Table 3 reports the proportions of successful forecasts. As for the two **satisfactory** trading policies, the number of successful bullish forecast is greater than the number of failed bullish forecasts. Unfortunately, the number of successful bearish forecasts is not greater than the number of failed bearish forecasts.

Table 3 – Nonlinear trading policies versus passive policies. Proportions of successful forecasts for the years 1999-2013 and four different pairs of parameters δ and *d*.

δ	*d*	$\frac{\text{S\&P up}}{\text{call up}}$	$\frac{\text{S\&P down}}{\text{call down}}$	# calls	$\frac{\text{successful call}}{\text{call}}$
11	6	$\frac{147}{253} = 58{,}1\%$	$\frac{35}{86} = 40{,}7\%$	339	$\frac{147+35}{339} = 53{,}7\%$
18	12	$\frac{94}{151} = 62{,}3\%$	$\frac{24}{55} = 43{,}6\%$	206	$\frac{94+24}{206} = 57{,}3\%$
12	7	$\frac{114}{222} = 51{,}4\%$	$\frac{30}{89} = 33{,}7\%$	311	$\frac{114+30}{311} = 46{,}3\%$
27	10	$\frac{61}{112} = 54{,}5\%$	$\frac{11}{24} = 45{,}8\%$	136	$\frac{61+11}{136} = 52{,}9\%$

An insightful term of comparison is provided by Brown *et al.* (1998), who test the forecasting power of the Dow Theory, summarised in Reuters Ltd. (1999, sect. 1). They examine all editorials published by William Peter Hamilton, second editor of the *Wall Street Journal*, who succeeded Charles Henry Dow, founding editor. They assume monthly trading and compare a trading policy based on such editorials with a passive policy based on the S&P 500 stock index. The former may call for a long position in S&P 500 stock index, or a riskless position that earns interest, or a short position in the S&P stock index, which is explained in Section 4.3. The former results in a higher portfolio value until 1926.

As shown in Table 4, where use is made of logarithmic returns in accordance with our setting, the former outperforms the latter over the years 1903-1929, displaying a slightly lower annual mean return and a much lower standard deviation.

Table 4 – Hamilton's trading policy versus passive policy. Reward to risk ratios $RR_{trading}$ and $RR_{passive}$ for the years 1903-1929; estimates based on Brown *et al.* (1998, Table II).

$\bar{r}$	σ	$RR_{trading}$	$\bar{r}_{SP}$	σ_{SP}	$RR_{passive}$
9,18%	8,24%	1,11	9,77%	11,24%	0,87

The number of switches is not reported in Brown *et al.* (1998); as mentioned earlier, infrequent switches curb commissions and fees due to trading.

The proportions of successful forecasts are

$$\frac{\text{S \& P up}}{\text{call up}} = \frac{74}{130} = 56{,}9\%\,, \quad \frac{\text{S \& P down}}{\text{call down}} = \frac{36}{54} = 66{,}7\%\,, \quad \frac{\text{successful call}}{\text{call}} = \frac{110}{184} = 59{,}8\%$$

Notably, the divergence-based procedure displays only slightly better proportions of successful bullish forecasts and much worse proportions of successful bearish forecasts; in spite of this, it results in a better performance, as it is such that $\bar{r} > \bar{r}_{SP}$ and $\sigma < \sigma_{SP}$.

Brown *et al.* (1998) also carry out an **event study** on the Dow Jones Industrial Average, i.e. another stock index, drawing the conclusion that Hamilton's trading policy takes advantage of **inertia** patterns. More precisely, they consider windows of 81 days: 40 days before the publication of each editorial and 40 days after it. Calls up are followed by a 1,5% price increase on average, calls down are followed by 1,74% price decrease on average, whereas neutral calls are followed by 0,21% price increase on average. Moreover, bullish forecasts tend to reflect recent upward trends in the stock index, whereas bearish forecasts tend to reflect recent downward trends.

4.2 Montecarlo tests

The forecasting power of the divergence-procedure is now tested against the null hypothesis that the S&P 500 stock index follows a random walk, the population moments of the daily logarithmic returns being constant. To do so, our **historical period** 1999-2013 is divided into 15 subperiods, each matching a calendar year. As a consequence, the number of daily logarithmic returns ranges from 250 to 253 per year.

More specifically,

- for each year under examination, the divergence-based procedure is applied to actual logarithmic returns. Therefore, different pairs of parameters δ and d are considered, eventually obtaining a grid of reward to risk ratios as in Figure 1. Both S & P %, the percent weight of white areas, and S & P max, the maximum reward to risk ratio, are reported in Table 5, where each value of S & P max is matched by a pair of parameters δ and d;
- random logarithmic returns are generated in accordance with a **normal** white noise; 200 simulations are run, each spanning a time period of one year. For each simulation, the divergence-based procedure is applied to random logarithmic returns that are matched by a grid of reward to risk ratios as in Figure 1. Both the percent weight of white areas and the maximum reward to risk ratio are computed. According to our 200 simulations, the median of the former indicator is equal to 39,57, whereas the median of the latter indicator is equal to 1,80.

Table 5 – Percent weight of white areas S & P % and maximum reward to risk ratio S & P max for the years 1999-2013. Each value of S & P max is matched by a pair of parameters δ and d.

year	S & P %	S & P max	δ	d
1999	38,05	2,19	25	6
2000	41,10	1,47	29	22
2001	50,23	1,56	26	5
2002	67,71	1,50	23	10
2003	1,52	2,39	10	8
2004	39,27	2,56	22	7
2005	15,53	2,02	13	5
2006	49,62	3,14	25	5
2007	19,48	1,37	27	10
2008	65,45	1,18	18	14
2009	17,35	2,66	30	8
2010	56,62	2,27	26	6
2011	73,06	1,22	11	9
2012	54,79	2,04	29	5
2013	9,74	2,41	29	19

Our preliminary null hypotheses are that

- the median of S & P % is different from 39,57;
- the median of S & P max is different from 1,80.

The median is not very dependent on extreme values and appropriate for skewed distributions. Let $N = 15$ be the total number observations and N_{median} be the number of observations lower than either 39,57 or 1,80. According to the non-parametric median test

(Anderson and Finn, 1997), our preliminary null hypotheses are rejected with a 95% confidence level if $\left|N_{median}/N-0{,}5\right|>0{,}98/\sqrt{N}$.

Notably, when performed by Strozzi and Zaldìvar (2005, sect. 3.1), the non-parametric median test results in the rejection of the null hypothesis of random walk. Unfortunately, both our preliminary null hypotheses are not rejected. Indeed, we get

$$\left|7/15-0{,}5\right|=0{,}03<0{,}98/\sqrt{15}=0{,}25 \quad \text{and} \quad \left|6/15-0{,}5\right|=0{,}10<0{,}98/\sqrt{15}=0{,}25$$

for the former and latter test, respectively. Possible remedies are listed in Section 5.

4.3 Possible extensions of nonlinear trading policies

Two promising extensions are considered in the sequel. As both extensions are beyond the scope of this paper, they are left to future research.

On the one hand, the **forecasting power** of the divergence-based procedure might be improved by the use of a threshold *L*, possibly complemented with the allowance for short positions. Keep in mind that a short position makes sense when stock prices are expected to fall; it can be taken by temporarily lending stocks from a financial intermediary and selling them on the open market. Accordingly, for $\Delta^2 V_t<-L$ a bullish forecast is made and a long position is taken in the **S&P 500** stock index, for $-L\le\Delta^2 V_t\le L$ no forecast is made and cash is idle, and for $L<\Delta^2 V_t$ a bearish forecast is made and a short position is taken in the **S&P 500** stock index.

On the other one, real-time forecasts might be made by relying on a moving data sample and performing the following calculations at time *t*

$V_{t-3\delta}$	$V_{t-2\delta}$	$V_{t-\delta}$	
$t-3\delta$	$t-2\delta$	$t-\delta$	t

1) a satisfactory triplet (δ,d,e), where $d<\delta$, is chosen by using the ratio

$$\frac{\text{annualised mean return}}{\text{annualised standard deviation}}$$

as the decision criterion and taking a countor plot into consideration;

2) $V_{t-\delta}$ is computed at time $t-\delta$;

3) $V_{t-2\delta}$ and $\Delta V_{t-2\delta}$ are computed at time $t-2\delta$;

4) $V_{t-3\delta}$, $\Delta V_{t-3\delta}$, and $\Delta^2 V_{t-3\delta}$ are computed at time $t-3\delta$; a trading decision is taken, the index fund being preferred to cash for $\Delta^2 V_{t-3\delta} < 0$, otherwise cash is preferred to the index fund;

5) the time step is updated and made equal to δ so that both a subsequent forecast can be made and a subsequent trading decision can be taken. Notice that the time step is not constant with time.

Real-time forecasting is both theoretically sounder and more likely to be operationally useful.

5. Conclusions

The **efficient stock market hypothesis** is common sense to some extent, as it states that earning a lot of money is very hard. Indeed, becoming a skilful trader or investor is very hard, as striking a balance between greed for quick profit, fear of a serious loss, and regret about missed opportunities is a real challenge. In addition, statistical analysis is not helpful for free riding, i.e. the resort to professional traders and investors. Actually, too long a data sample is needed to check with a 95% confidence level whether a good performance and its persistence are due to skill or luck. Nonetheless, informational inefficiencies can be removed only by the action of skilful traders and investors, which is a motivation for applied research.

This work has focused on the trading on the S&P 500 stock index. More precisely, it has expanded on Strozzi and Zaldìvar (2005) as well as Strozzi and Zaldìvar Comenges (2006), who developed a divergence-based procedure for trading in foreign currencies and tested it successfully on high-frequency data about the exchange rates between the US dollar and 18 other currencies. In this work, daily data have replaced high-frequency data, the data sample spanning 15 years rather than one year. Moreover, the performance of the nonlinear trading policies has been measured in terms of a reward to risk ratio and the robustness of our numerical results has also been checked in a few subperiods. Both features are important in financial economics as well as business practice.

According to our numerical results, a careful selection of reconstruction parameters results in a promising performance. The nonlinear trading policies outperform their benchmarks, i.e. some passive portfolios, displaying higher annualised mean returns, i.e. higher final portfolio values,

as well as lower annualised standard deviations. Notably, they don't require a high number of switches.

Unfortunately, Montecarlo tests have been failed in contrast with Strozzi and Zaldìvar (2005, sect. 3.1). Notably, bootstrapping tests are altogether in favour of the Dow Theory (Brown *et al.*, 1998, sect. 3). However, the present implementation of the divergence-based procedure doesn't allow for a short position in the S&P stock index, as in the case of the Dow Theory (Brown *et al.*, 1998). Moreover, our assumption that cash doesn't earn interest could be too conservative. Finally, alternative null hypotheses for Montecarlo or bootstrapping tests could be taken into account. Therefore, additional and future research is required. At any rate, the present off-line implementation of the divergence-based procedure is not suited for an operational use; however, an on-line extension is feasible and has been examined in detail. Needless to say, a portfolio manager should base his/her trading decisions on a battery of indicators rather than on a single indicator, including **fundamental** indicators (about companies, industries, and economies) as well as **technical** indicators. A case in point is the quantitative approach sketched in Haugen (1997, chapt. 6). Actually, considerable numbers of traders and investors make use of both **fundamental** and **technical** analysis Reuters Ltd. (1999, sect. 1).

More generally, additional and future research might also ascertain whether the divergence based procedure can be applied successfully to other financial time series. Keep in mind that a deeper understanding of financial time series and their properties may come in useful when it comes to the regulation of financial markets, especially by taxation, or possibly by restricted access to qualified traders and investors, such as chartered financial analysts.

Acknowledgements

To the memory of José Manuel Zaldìvar Comenges, who was the first to realise that the divergence-based procedure presented in this paper could carry over to forecasting financial time series.

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Sommario

Si applica un procedimento, fondato sulla divergenza, alla negoziazione dell'indice azionario S&P 500. Tale procedimento è stato precedentemente applicato con successo alla negoziazione di valute estere. La prestazione viene confrontata con un portafoglio di riferimento, vale a dire un portafoglio passivo che replica l'indice azionario S&P 500; la sua robustezza è pure verificata in alcuni sottointervalli temporali. Secondo la nostra evidenza numerica, si possono ottenere, in tutti i sottointervalli temporali, maggiori rendimenti logaritmici medi, ossia maggiori montanti finali, e minori deviazioni standard. Tuttavia, elementari test Montecarlo non vengono superati. Si considera pure la possibile estensione in linea della presente applicazione fuori linea, in quanto essa è più adatta a un impiego operativo.

Abstract

A divergence-based procedure is applied to trading in the S&P 500 stock index. Such a procedure has previously and successfully been applied to trading in foreign currencies. Performance is tested against a benchmark, i.e. a passive portfolio replicating the S&P 500 stock index; its robustness is also checked in a few subperiods. According to our numerical evidence, higher annualised mean returns, i.e. higher final portfolio values, as well as lower annualised standard deviations can be obtained in all subperiods. However, basic Montecarlo tests are failed. The on-line extension of the present off-line implementation is taken into consideration, as it is more suited for an operational use.

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