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Sovereign Credit Risk in the Eurozone*

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A MODEL FOR PUBLIC DEBT SUSTAINABILITY AND SOVEREIGN CREDIT RISK IN THE EUROZONE

*Marcello Esposito**

1. Introduction

The sovereign debt crisis in Europe renewed the interest of scholars and policymakers for disentangling the “fundamental” component of the interest rate paid on government bonds from the “speculative” one. The latter is considered particularly dangerous because it can generate self-fulfilling expectations about the default of the obligor.¹ The Outright Monetary Transactions (OMT) program by the ECB has been explicitly structured to avoid the impact on the spread caused by unjustified speculative attacks or by a confidence crisis².

So far the economic literature has tackled the issue of determining the “fundamental” component of the sovereign spread mainly on empirical grounds, by regressing the actual spread on some variables reflecting the fundamentals of a country³ or by relying on qualitative analysis of multiple indicators, analyzed historically and with respect to other countries. The template is the Debt Sustainability Analysis (DSA) developed by the IMF (2011). The objective of the DSA is not to define the fundamental spread that a sovereign issuer should pay. But from the DSA one can form groups of countries with similar debt fundamentals and compare the interest paid on their bonds in order to detect anomalies.

On theoretical grounds, some recent contributions have taken up the approach originated by Calvo (1988), analyzing the choice of a government between servicing its debt and defaulting.⁴ These models are able to identify, in principle, the level of the debt/GDP ratio beyond which a government is not able to raise more money in financial markets. However, they rely on social loss functions very difficult to estimate empirically.

The purpose of this paper is to fill the gap between those two strands of literature, by providing a simple stochastic generalization of accounting models for debt sustainability. From

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this, we derive a model for sovereign credit risk that can be easily taken to the data and verified. As in Calvo, we obtain the possibility of multiple equilibria and then a framework for analyzing policy intervention in the government bonds market.

We will proceed as follows. In paragraph 2, we will review the algebra of public debt in order to gain insights into the issue of public debt sustainability. We will use the “Maastricht” parameters to find a way of determining a critical limit for public debt sustainability. This will be the crucial ingredient for determining the “default probability” of a sovereign issuer. In paragraph 3, we will resort to an “external rescue” model to determine at which conditions a supranational agency (say the Troika⁵) will offer a financial assistance program to a sovereign country whose debt becomes unsustainable. This will be the crucial ingredient for determining the “loss given default”. Finally, in paragraph 4, we will put the things together in order to determine the credit spread paid by a sovereign obligor. As we will show, our model produces a variety of equilibria, encompassing the previous literature on multiple equilibria and without the need to resort to arbitrary loss functions. Our approach allows for studying the dynamic properties of the equilibria and gain some insights into the “good” and “bad” nature of the single equilibrium points from the ECB perspective.

2. Determining the critical limit for debt sustainability and the consequences of breaking that limit

As Ley (2010) noted, the algebra of public debt accounting identities provides a source for insight to identify risk to sustainability, but the difficulty in individuating a critical level is that it depends on a very large number of factors. Moreover, most of these factors interact between themselves, sometimes in a non-linear way. Consider for example the fact that the most important contribution to a ballooning public deficit often comes from interest rate expenditures on government bonds. The interest rate is the sum of a risk-free component and a credit-risk component. The credit risk component depends on the default probability and the loss given default. It is not difficult to find historical examples of self-fulfilling expectations: higher interest rates feed back into higher debt that feeds back in turn into higher interest rates. The degree of interdependence is determined by the degree of openness of the economy to international capital movements, the percentage of public debt held by foreigners and/or denominated in a foreign currency, etc etc.⁶

The typical government’s solvency condition states that the net present values of primary surpluses and debt obligations have to be equalized. As it is known, in a deterministic environment for the solvency condition to be satisfied it is not necessary that all the public debt be paid back, but it is sufficient that the debt/gdp be stabilized (Eisner, 1994). In theory, for

every level of debt/gdp, it is always possible to generate a constant primary surplus such that the ratio is stabilized and the solvency condition satisfied. However high primary surpluses can generate distortionary effects on the economy and implies always strong redistribution policies among the population. For these reasons, one of the tenet of the debt sustainability literature is that the higher the required primary surplus the lower the likelihood that the public debt is sustainable.

We define the “critical” level of debt/gdp, d_t^* , as the level above which debt repudiation occurs with probability 100%. The default probability of a public obligor depends on: i) the expected path of the primary surplus/deficit of the government; ii) the initial level of debt/GDP ratio, and iii) the expected GDP growth. These elements determine the probability that the debt/GDP will eventually reach the critical level d_t^* . Such probability, together with the assumed loss given default, determines the interest rate spread paid over the market risk free rate.

In the most recent theoretical papers, the critical debt/gdp level is derived by a “social” loss function, measuring the losses caused by the distortionary effects of high primary surpluses, and modelling the strategic game between the sovereign obligor and the investors. The problem is that the social loss function is unobservable and its parameters and form can’t be estimated.

At the moment, we are not aware of an alternative approach to determining the critical debt/gdp level. However, the peculiarities of the Eurozone can help developing an intuition of how one could derive an approximate localization of the critical area for the debt/gdp level.

2.1 The Eurozone peculiarities and the determination of a critical area for the sustainability of the public debt

The Stability and Growth Pact (SGP) in essence consists of fiscal monitoring of member states and the issuing of policy recommendation to ensure a fiscal and financial stability inside the European Union. If a member state breaches the maximum limit for government deficit and debt, an Excessive Deficit Procedure (EDP) can be declared. We don’t want to dig into the arcana of the “preventive arm” and the “dissuasive arm”, the procedures in which the surveillance activity is articulated. We just want to underline that the SGP aims for each country to stay within the limits on government deficit (3% of GDP) and debt (60% of GDP). In case debt is above 60%, SGP asks for fiscal policies such that the debt level each year declines towards 60% for 1/20 of the distance. Normal people would interpret the “debt rule” to imply a 20 year convergence to the 60%. In reality, the interpretation given by the Commission looks more like the first policy implementation of the Achille’s paradox. Working out the algebra, the Commission seems to be happy with fiscal consolidation plan that, starting with levels above

120%, cut heavily the debt at the beginning and then slowly bring the debt below 90% in 20-30 years.

Let's not follow the mathematical byzantine of the Commission and let's assume for simplicity that the target is to converge to 60% in 30 years. The fiscal consolidation plan consists of the path of primary surpluses, S , that the country has to follow in the future. In theory, the surplus can be as high as necessary to converge to the 60%. In practice, there are limits to the surplus that can be run without impairing in a counterproductive way the GDP growth rate and the necessary political consensus.

We will make the heroic assumption that a democratic government can impose to its country a 2% primary surplus for 30 years. We will also assume that the European Commission would tolerate a fiscal consolidation plan that consists in reaching a 90% (instead of 60%) target in 30 years. The choice of 90% is arbitrary but it is justifiable on the ground that it is recognized as a threshold for the debt distortionary effects on growth.⁷

If 90% is the debt/gdp target in 30 years, we can use the algebra of public debt accounting in order to derive a range of initial debt/gdp values coherent with this target. In the appendix we show how to calculate the maximum level of debt today that is coherent with reaching a certain target after N years. In our case (see Table A.2), the range is 119%-150% for the debt/gdp today, depending on the assumptions that we make about the real growth rate of the economy and the real rate on government bonds. Based on the historical experience with the countries that asked for a financial assistance from the European partners, a common narrative on the market is that 140% is the tipping point. Since this value is coherent with the range, for the sake of simplicity, we will use it in the numerical examples below.

2.2 The “external rescue” model

In our conceptual model we have two agents that are external with respect to the country in financial difficulty. The first is the debt agency (the Troika), whose objective is to reduce the burden of fiscal consolidation, providing loans to the country at an interest rate which is lower than the one justifiable by public finances' fundamentals. The second is a central bank (the ECB) whose objective is to avoid a self-fulfilling default caused by arbitrary increases in interest rate but that can't take on purpose the credit risk of the issuer and implicitly bail-out the debt-holders.

What happens when a country reach the critical level? There is no way to trigger an automatic default. In theory, the market could irrationally price a negative interest rate and then the debt might be sustainable also at very high debt/gdp levels. And it is also true that a default could be triggered well below the critical level if the “market” loses confidence in the

capabilities of the sovereign issuer to repay its debt. It is easy to understand that the expectation of a default can become self-fulfilling also at very low debt/gdp levels, see Cole-Kehoe (2000).

In the European context, however, a country has the opportunity to ask to the other member countries and the IMF for financial assistance, through EFSF/ESM and under the surveillance of an informal international agency, the so-called Troika. The objective of the agency is not only to avoid a sunspot-driven default of the type described in Cole-Kehoe (2000), but also to bring the country debt back to 60% in 30 years. The agency might ask for an initial haircut on outstanding bonds, in order to start the fiscal consolidation program at a more manageable level.

2.3 financial assistance, investors' bail-in, and the emergence of a sovereign spread

If the help of the international agency comes at no price for the debt-holders, the government bonds are always considered risk-free. In our conceptual model of the Eurozone, however, we assume that the agency, before intervening, asks for a partial bail-in of the existing debt-holders, through a haircut on existing bondholders.

Why the agency should ask for a bail-in? As we said before, in theory, the surplus can be as high as necessary to converge in 20-30 years to the 60-90%. In practice, there are limits to the surplus that can be run without impairing in a counterproductive way the GDP growth and then the capability of reaching the comfort zone. Especially, if the responsibility of the convergence program rests on a technocratic structure that is not the direct expression of the democratic will of the people resident in the country.

So the agency might consider viable a lower primary surplus than the one that a sovereign and democratic government might impose on its citizens. The agency might also be more risk-averse than the previous government and so it might use conservative estimates about future economic outlook, in order to minimize the risk of organizing another rescue operation in the near future.

In the numerical examples that will follow, we will then assume for the agency's plan a primary surplus of 1%, which is 50% lower than the one assumed for deriving the critical debt level for a democratic government. In table A.1 we calculate a range of initial debt/gdp levels that are coherent with a 60% target in 30 years. The result is a 70%-90% range. For the sake of simplicity, we will pick-up the upper limit of the range, i.e. 90%, as the starting level of the fiscal consolidation plan under the guide of the agency. This implies an overnight reduction, via haircut, of the debt/gdp to 90% in case of a request for a financial assistance. The possibility of such a "credit" event generates the sovereign spread

2.4 mapping the debt/gdp space: an example

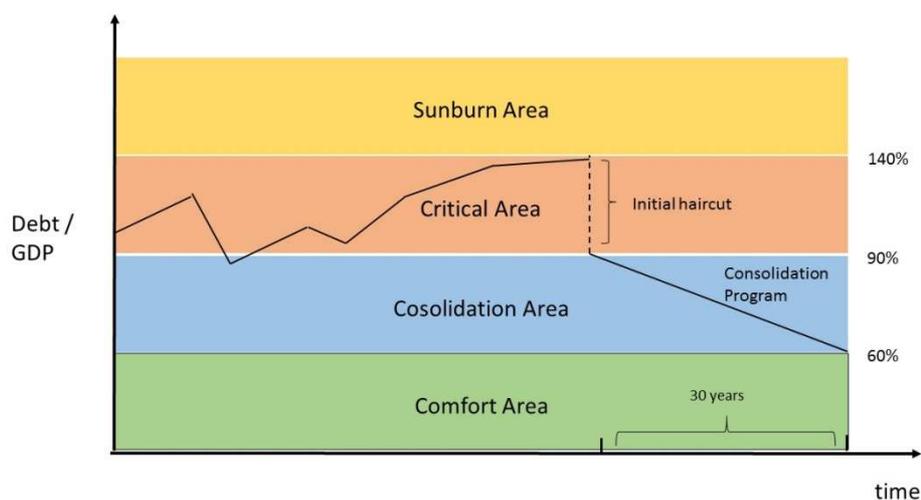
Summing up our conceptual model, we have three crucial values for the debt/gdp. The lowest is 60%, corresponding to the Maastricht criteria, and the highest is 140%, corresponding to the critical value for debt sustainability. Below 60% we are in a “comfort area”, where the country has full control of its public finances and can increase debt, if the government thinks that it is necessary. Over 140% we enter an area where a self-fulfilling default can’t be avoided. Inside the minimum and the maximum, we have 90%. As we explained in section 2.3, if the country asks for financial assistance, the debt agency will impose a haircut on the nominal value of outstanding debt such that it can start the financial consolidation plan at the 90%.

We will call the area comprised between 60% and 90% as the “consolidation area”. Inside this area, there is no risk of debt default but the country has to reduce its debt and then has no full control of its public finances. The area between 90% and 140% is the “critical area”, because if the country ask for assistance, the debtholders will suffer a loss. The area above 140% is the “sunburn area”, in honor to the “sunspot” literature, where a self-fulfilling default is unavoidable because the central bank can’t intervene to stop a speculative attack.

Inside the critical area the spread arises and increases as the debt/GDP moves toward the upper limit. The spread takes in fact into account the probability of entering the sunburn area and the country asking for financial assistance. In this case the level of debt is restructured and reduced to the lower limit of the critical area.

As an example in Figure 1 we represent the dynamics of our conceptual model. On the vertical axis we have the debt/GDP ratio and on the horizontal axis the time. The space is divided in 4 areas, depending on the level of debt/GDP.

Figure 1 – The map of debt/gdp space



3. Pricing credit risk on government bonds

In order to calculate the credit spread paid by a bond we need to calculate the Probability of Default (PD) and the Loss Given Default (LGD). For simplicity of notation, let's assume that the GDP real growth rate is zero and then we can use the capital letter D to denominate the Debt/GDP ratio without making confusion with differential operators and with the traditional notation of public debt dynamics.

Keeping in mind the intuition described in the conceptual model of section 2., we assume that there is an upper bound to the debt/gdp process, the "critical" level: \bar{D} . When debt/gdp touches the "critical" level, bonds are restructured and their nominal value is immediately cut to a level: \underline{D} . This part of the model produces the LGD, which is equal to: \bar{D} / \underline{D} .

The PD depends on the probability that the debt/gdp hits the upper bound. This event depends on many factors: the actual level of the debt/gdp, the real growth of the economy and the impact of the fiscal policy on the economy's nominal growth rate, the maturity structure of the debt, the currency denomination of the debt, etc etc. However, we will focus on just two parameters: the initial level of the debt/gdp and the interest rate.

Since the probability of hitting the barrier depends on the dynamics of the deficit, which depends in turn on the credit spread, we have a multiplicity of possible levels of interest rates as solutions to our model. For the sake of simplicity, we will assume that the public debt is constituted by a single nominal bond with a fixed coupon and a given maturity (say, equal to the average maturity of the debt outstanding). The interest rate on the government bond, r , is the sum of the risk-free rate, r_{free} , and the spread, r_{spread} .

$$r = r_{free} + r_{spread}$$

Since the country is assumed to be "small", the risk-free rate dynamics is exogenous whereas the spread is endogenous.

3.1 the stochastic dynamics of the debt

We will assume that the debt evolves in a first approximation according to a geometric Brownian motion, whose properties have been extensively studied in Finance⁸:

$$dD/D = (x + r_{spread})dt + \sigma dz$$

Where $x = r_{free} - s$, s is the primary surplus, and σ is the volatility of the debt growth rate. The probability of the debt to hit the barrier, \bar{D} , after T years is equal to:

$$PD = N\left(\frac{\ln(D_0/\bar{D}) + \left(x - \frac{1}{2}\sigma^2\right)T + r_{spread} * T}{\sigma\sqrt{T}}\right)$$

Where $N(\cdot)$ denotes the standardized Normal distribution and D_0 is the initial value of debt.

3.2 the “equilibrium” credit spread and its dynamics

The equilibrium credit spread for a bond maturing in T years is the product of the PD and the LGD, divided by T :

$$r^* = PD \frac{LGD}{T}$$

Where the LGD is equal to the ratio between the level of debt today and the level of debt implicit in the bail-in imposed by the agency in case of a request for financial assistance at time T : D_0/\underline{D} .

The PD depends, among other things, on the credit spread. The credit spread is in fact a component of the interest rate paid on government bonds.

The equilibrium credit spread is then the solution to the following equation:

$$r^* = \frac{D_0/\underline{D}}{T} * N\left(\frac{\ln(D_0/\bar{D}) + \left(x - \frac{1}{2}\sigma^2\right)T + r^*T}{\sigma\sqrt{T}}\right)$$

The equation is non-linear and we could have situations with multiple equilibria or no equilibrium at all. Moreover, in case of multiple equilibria, we need to understand which ones are stable and which ones are desirable, i.e. “good” or “bad”, from the point of view of financial stability.

In order to understand the dynamic properties of the equilibria, we will resort to the economic interpretation of the iterative method for finding a numerical solution to the equation. Specifically, we can interpret the numerical method as the equivalent of a “*tatonnement*” process of price discovery. Let’s then introduce two new variables, the Market_Spread and the Fair_Spread. The Market_Spread is the actual credit spread priced by the market. The Fair_Spread is instead the theoretical spread, derived by calculating the probability of default for a given Market_Spread. In equilibrium, the Market_Spread has to be equal to the Fair_Spread.

The Fair Spread is a function of the product: $PD * LGD$, where PD is the Probability of Default, i.e. the probability of the public debt hitting the barrier (say 140%), and LGD is the Loss Given Default, i.e. the haircut (say 40%) imposed by the international agency in order to offer a financial assistance program. The probability of default depends on the “Market Spread”. We will be in equilibrium when the Fair Spread and the Market Spread are equal. In order to find an equilibrium value for the spread, we can start the discovery process with an arbitrary value of the Market_Spread and derive the Fair_Spread. If the two are not equal, we go to the next iteration step and we assume that the Market_Spread is equal to the Fair_Spread calculated in the previous step. Then, we calculate the new value of the Fair_Spread and we test if the result is identical to the Market_Spread. The iteration carries on until we find an equilibrium or we reach one of the two corners of the admissible universe, i.e. 0% or 100% default probability. It is easy to understand that this is equivalent to the numeric method used to solve the non-linear equation.

An approach similar in spirit to the one described above is in Gross (2012). The main difference is in the fact that Gross (2012) derives the function linking the Fair_spread to the Market_spread from the creditors-obligor strategic interaction, given that higher interest rate charged by the market increases the debt service burden and thus the temptation of the obligor to default. The feedback loop in Gross (2012) depends on the interaction between the costs of default and the incentive to default as subsumed by a social loss function. In our model, in contrast, there is no strategic game between debtor and creditors. Despite these differences, his *tatonnement* process for the search of an equilibrium is very similar to phase diagram dynamics implicit in our model.

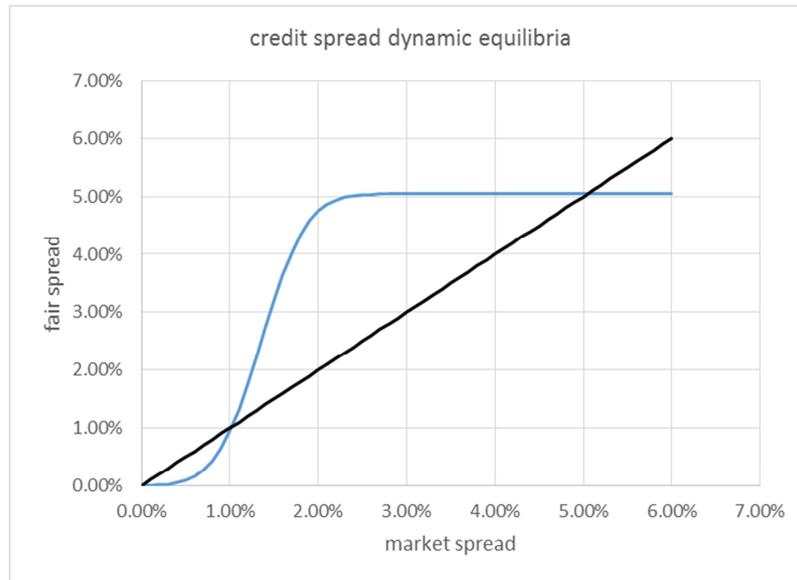
Our model can produce at maximum 3 possible equilibria. With low level of debt, the equilibrium is unique, with a fair spread equal to zero. With high level of debt, the equilibrium is unique too, but it implies a fair spread corresponding to a default probability equal to 100%. In between, we have the possibility of 3 equilibria. In this case, the lower and the upper equilibria are corner solutions, corresponding to 0% default probability and 100%. The intermediate equilibrium is instead coherent with a default probability lower than 100%.

In order to understand the dynamics, let's consider a graphical representation of the equation, with the following parametrization of the economy. The debt is represented by a single bond maturing in 6 years time, the volatility is 1%, the level of the risk-free rate is 0.5% as the growth rate of the economy. The actual level of the public debt is 133%, the barrier is equal to 140%. If the maximum acceptable debt/gdp level for the agency is 90% and the nominal growth rate of GDP is 0.5% per annum, then over a 6 year time horizon:

$$LGD = D_0/92 \text{ and then the LGD is } 30\%.$$

In the figure below we depict the Fair Spread as a function of the Market Spread. The solutions to the equation are where the function crosses the 45 degree line.

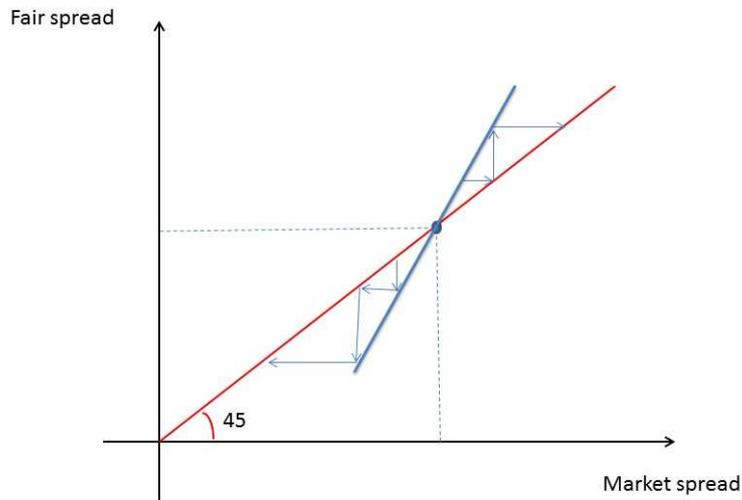
Figure 2 – Determining the equilibrium spread



As we can see, in our specific example we have multiple equilibria. The lower and the higher solutions are corner solutions, corresponding to the 0% and 100% probability of default, respectively. The intermediate solution is instead the one that is coherent with a positive but less than 100% probability of default.

The figure can be interpreted as a phase diagram, where the solution is reached through a *tatonnement* process of price discovery. The equation can be linearized around the equilibrium point by using an approximation to the Normal cumulative distribution. For example, if we linearize around the intermediate equilibrium point we see that it is unstable. Instead, it is quite easy to see that the “corner” equilibria are both stable.

Figure 3 – The equilibrium dynamics of the credit spread



3.3 The quality, “good” and “bad”, of the equilibria

The dynamic analysis of the system allows to gain some insight in the “attractiveness” of the equilibria from the perspective of a special actor in the Eurozone debt market, the ECB. The ECB can’t monetize the debt of member countries but can intervene if the financial stability or the existence of the euro is in danger. For this reason, during the sovereign debt crisis, the ECB devised the OMT program as an extraordinary tool for contrasting speculative attacks on the debt of specific member countries. In order to avoid the critic of debt monetization, the ECB had to state that with the OMT the objective is to bring the spread to its “fundamental” value, purging the market only from speculative forces increasing the cost of servicing the debt without justification.

From the dynamics of the system analyses in the previous section, we understand the importance of the OMT. The intermediate equilibrium is, in fact, unstable and if speculative forces bring the market spread to the right, then we enter a dynamics of self-fulfilling expectations ending with the default of the issuer. At the same time, we don’t have a unique solution to the equation and then we don’t have a unique “fundamental” equilibrium. So, what is the value of the spread that the ECB should target?

We can obviously discard the highest equilibrium, because it is coincident with 100% probability of default. We remain with the zero equilibrium and the intermediate one. It would be rational for the ECB to target directly the zero equilibrium. In this way it minimizes the possibility of losses on its bonds’ portfolio. However, this could expose the ECB to “political” criticism and there is an alternative strategy that can save the balance sheet of the central bank

(i.e. minimize the amount of monetary base created). This alternative is the one of targeting a level of the spread which is a few basis point below the intermediate one. Once this level is reached, the market forces will make the last mile and bring the spread to zero.

4. Conclusions

The sovereign debt crisis in Europe highlighted the importance of the institutional set-up and the characteristics of the financial assistance program offered by international agents (the “Troika”, the ECB) for understanding the credit risk of government bonds. In the context of Eurozone, where a member state has lost the possibility of money financing, we derive the sustainability conditions for public debt on the basis of the constraints imposed by the Maastricht Treaty on public finances.

We obtain an upper limit for public debt/gdp above which the country is exposed to the risk of a self-fulfilling default. In order to avoid a self-fulfilling default the government can ask for a financial assistance by a supranational agency (the Troika). The Troika can provide the assistance only if it is possible for the country to respect the Maastricht parameters over a long but not infinite period of time. In our model the supranational agency, before intervening, asks for a partial bail-in of the existing debt-holders, through a haircut on outstanding government bond. This is due to a higher risk-aversion (than the one of the domestic government) and the consciousness that a technocratic institution is not able to run a primary surplus as big as a democratic government. The agencies in fact want to minimize the possibility of asking their stakeholders for a new financing round in case the fiscal assistance program fails to reduce the public debt level to the agreed target. In our model there is a role also for the central bank like the ECB that can't monetize the public debt but can avoid a sunspot default by keeping the “spread” near its fundamental value.

In our model we then derive endogenously the probability of default and the loss given default. We obtain the credit spread paid by a sovereign issuer by solving a non-linear equation with multiple solutions. Assuming that the market solves the equation through a “tatonnement” process, the dynamics of the spread can be represented through a phase diagram showing which solutions are stable and which ones are not. We show that this dynamic analysis is useful in gaining some insight in the functioning of the OMT program, devised by the ECB to avoid situations of self-fulfilling default expectations.

Appendix A. The algebra of public debt dynamics

The debt dynamics equation is:

$$D_{t+1} = D_t(1 + n_t) - S_{t+1}$$

where n is the nominal interest rate paid on government debt. Dividing by the level of nominal GDP, Y :

$$\begin{aligned} \frac{D_{t+1}}{Y_{t+1}} &= \frac{D_t}{Y_{t+1}}(1 + n_t) - \frac{S_{t+1}}{Y_{t+1}} \\ \frac{D_{t+1}}{Y_{t+1}} &= \frac{D_t}{Y_t} \frac{Y_t}{Y_{t+1}} (1 + n_t) - \frac{S_{t+1}}{Y_{t+1}} \\ d_{t+1} &= d_t \frac{(1 + n_t)}{(1 + y_t)} - s_{t+1} \end{aligned}$$

Eliminating the inflation rate:

$$\begin{aligned} d_{t+1} &= d_t \frac{(1 + r_t)}{(1 + g_t)} - s_{t+1} \\ d_{t+1} &= d_t(1 + z_t) - s_{t+1} \end{aligned}$$

Where:

$$z_t = \frac{(r_t - g_t)}{(1 + g_t)}$$

Targeting a specific debt level in a given time frame

Assuming that z is constant and s is equal to s_{max} , it can be obtained that after N years:

$$d_{t+N} = d_t(1 + z)^N - s_{max} \sum_{j=0}^{N-1} (1 + z)^j$$

Now, if d_{t+N} has to be equal to d_{min} (e.g. 60%), we can work out the maximum level of initial public debt coherent with this result :

$$d_t^* = \frac{s_{max} \sum_{j=0}^{N-1} (1+z)^j + d_{min}}{(1+z)^N}$$

For starting levels of debt that are higher than d_t^* , the target d_{min} can't be achieved (without a debt restructuring at the outset).

Determining a range of “critical” debt/gdp values

In order to understand if a certain debt level is sustainable, we need to specify a value for z term, i.e. a value for the real growth rate of the economy and a value for the real interest rate on government bonds. One can then understand the relationship between the economic cycle (affecting the real gdp growth) and the monetary policy of the core countries (affecting the real risk-free interest rate) in modifying the probability of default of highly indebted countries.

If we have the level of the risk-free interest rate and the maximum sustainable primary surplus, s_{max} , we only miss the GDP growth rate in order to work out the level of the debt/GDP at the end of the plan period for every initial level of the debt/GDP.

In the table below, under the hypothesis of a primary surplus equal to 2%, we calculate the maximum initial debt/gdp ratio coherent with reaching in 30 years⁹ the 60% target, for different values of the real interest rate and the gdp growth rate. We highlighted in brown the more likely range of values, coherent with a “golden rule” for real interest rates (higher than the real GDP growth). As one can see, the range goes from 71% to 90%.

Table A1- Maximum starting debt/GDP ratio coherent with a 60% final target to be reached in 30 years with a 1% deficit

		Maximum initial Debt/GDP coherent with a 60.0% target in 30 years (primary surplus=1.0%)						
		real interest rate						
		0.0%	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%
real GDP growth rate	-1.0%	70%	63%	56%	50%	44%	40%	36%
	-0.5%	80%	70%	63%	56%	50%	45%	40%
	0.0%	90%	80%	71%	63%	56%	50%	45%
	0.5%	102%	90%	80%	71%	63%	56%	50%
	1.0%	116%	102%	90%	80%	71%	63%	56%
	1.5%	131%	115%	102%	90%	80%	71%	63%
	2.0%	149%	131%	115%	102%	90%	80%	71%
	2.5%	169%	149%	131%	115%	102%	90%	80%
	3.0%	192%	169%	148%	130%	115%	102%	90%

In case we assume that the primary surplus is higher (2%) and the final target for the debt is higher (90%), the simulations change as in the table below.

Table A2- Maximum starting debt/GDP ratio coherent with a 90% final target to be reached in 30 years with a 2% deficit

		Maximum initial Debt/GDP coherent with a 90.0% target in 30 years (primary surplus=2.0%)						
		real interest rate						
		0.0%	0.5%	1.0%	1.5%	2.0%	2.5%	3.0%
real GDP growth rate	-1.0%	119%	106%	95%	85%	77%	69%	63%
	-0.5%	133%	119%	106%	95%	85%	77%	69%
	0.0%	150%	133%	119%	106%	95%	85%	77%
	0.5%	169%	150%	133%	119%	106%	95%	86%
	1.0%	191%	169%	150%	133%	119%	106%	95%
	1.5%	216%	191%	169%	150%	134%	119%	107%
	2.0%	244%	215%	190%	169%	150%	134%	119%
	2.5%	276%	243%	215%	190%	169%	150%	134%
	3.0%	313%	275%	243%	215%	190%	169%	150%

References

- Blanchard O., Leigh R. (2013), *Growth Forecast Errors and Fiscal Multipliers*, «IMF working paper», 13/1.
- Calvo G. (1988), *Servicing the public debt: the role of expectations*, «American Economic Review», 78(4), 647 – 661.
- Cole H., Kehoe T. (2000), *Self-fulfilling Debt Crises*, «Review of Economic Studies», 67, 91-116.
- Court of Justice of the European Union (2015), *Press Release, 2/2015*, 14 January 2015
- De Grauwe P. (2011), *The governance of a fragile Eurozone*, «CEPS Working Document», 346.
- De Grauwe P., Ji Y. (2013), *Self-fulfilling crises in the Eurozone: an empirical test*, «Journal of International Money and Finance», 34, 15-36.
- Di Cesare A. et al. (2012), *Recent estimates of sovereign risk premia for euro-area countries*, «Bank of Italy Occasional Paper», 128.
- Diamond D., Dybvig P. (1983), *Bank runs, deposit insurance, and liquidity*, «Journal of Political Economy», 91, 401-419.
- Easterly W. and S. Fischer (1990), *The Economics of the Government Budget Constraint*, *World Bank Research Observer*, 5:2, 127-42.
- Eisner R. (1994), *The Misunderstood Economy: What Counts and How to Count It*, Harvard University Press, Boston MA.
- European Central Bank (2012), *Introductory statement to the press conference of the President*, Sept. 6.
- Fink F., Scholl A. (2011), *A Quantitative model of sovereign debt, bailouts, and conditionality*, «University of Konstanz working paper series», 2011-46.
- Gros D. (2012), *A simple model of multiple equilibria and default*, «CEPS Working Document», 366.

- Herndon T., Ash M., Pollin R. (2013), *Does High Public Debt Consistently Stifle Economic Growth? A Critique of Reinhart and Rogoff*, Political Economy Research Institute, University of Massachusetts Amherst, http://www.peri.umass.edu/fileadmin/pdf/working_papers/working_papers_301-350/WP322.pdf
Retrieved May 2, 2013.
- Hull J. (2012), *Options, Futures, and Other Derivatives*, 8th edition, Pearson
- IMF (2011), *Modernizing the Framework for Fiscal Policy and Public Debt Sustainability Analysis*
- IMF (2013a), *Sovereign debt restructuring—Recent developments and implications for the Fund’s legal and policy framework*, «IMF Policy Paper.»
- IMF (2013b) *Greece: Ex Post Evaluation of Exceptional Access under the 2010 Stand-By Arrangement*, «IMF Country Report», 13/156
- Ley E. (2010), *Fiscal (and External) Sustainability*, Economic Policy and Debt Department, PREM, The World Bank
- Reinhart C., Rogoff K. (2010), *Growth in a Time of Debt*, «American Economic Review», 100(2), 573–78
- Roch F., Uhlig H. (2012), *The Dynamics of Sovereign Debt Crises and bailouts*, «University of Chicago working paper»

Notes

- ¹ The seminal contribution in this area was made by Calvo (1988), showing the existence of multiple equilibria for the sovereign bonds’ prices, due to the self-fulfilling nature of a confidence crisis in the obligor’s solvency. The Calvo argument parallels the well-known Diamond-Dybvig (1983) model.
- ² See ECB (2012). The Advocate General of the Court of Justice of the European Union (2015) gave an opinion confirming that the OMT are compatible in principle with the Treaties of the European Union
- ³ See Di Cesare *et al.* (2012) and the literature reviewed there.
- ⁴ See Gros (2012), De Grauwe (2011), De Grauwe and Ji (2013).
- ⁵ In Europe, the term Troika denotes the informal body controlling the implementation of the rescue programme for the countries belonging to the Eurozone (Greece, Portugal, and Ireland). It is formed (up to 2015) by the European Commission, the IMF and the ECB.
- ⁶ The IMF (2013b) recognizes the necessity to develop a framework to better accommodate the circumstances of monetary unions.
- ⁷ This is the famous result of Rogoff-Reinhart (2010) which has been criticized by Herndon, Ash, Pollin (2013) for some errors in the econometric estimates.
- ⁸ See Hull (2012)
- ⁹ In the Fiscal Compact the debt rule should be to converge in 20 years. In reality, we have seen that the beginning of the debt reduction process has been already postponed by some years for many countries. Moreover, the rule is stated in an ambiguous way so that it has been interpreted by the European Commission in a flexible way, similar to the Achilles and the turtle paradox. Taking into account those considerations, 30 years seems more appropriate for the European context.

Sommario

La determinazione del valore dello spread sui titoli di Stato in base ai "fondamentali" economici dell'emittente sovrano rappresenta un problema di particolare rilievo nell'Eurozona. Gli eventuali interventi della BCE sul mercato dei titoli di Stato sono infatti giustificabili in base alla lettura corrente dei Trattati solo se lo "spread" pagato dall'emittente sovrano è superiore a quello che i "fondamentali" prevedono. La complicazione nel determinare quale è il livello "giusto" dello spread dipende dal fatto che la sostenibilità di un certo debito pubblico è funzione di parecchi fattori, tra cui il livello stesso dei tassi d'interesse pagati. Questa sorta di circolarità tra sostenibilità e tassi d'interesse è la sorgente primaria di complessità. Questo lavoro illustra una possibile soluzione all'interno di un contesto semplificato che riprende alcune peculiarità dell'assetto istituzionale europeo: assenza di possibilità di monetizzazione del debito, i famosi parametri 3%-60% di Maastricht, la disponibilità di un piano di aiuti finanziari subordinato all'accettazione di un piano di consolidamento fiscale. Nel nostro modello otteniamo la possibilità di equilibri multipli per il valore dello spread, le cui proprietà dinamiche possono essere derivate dalle assunzioni probabilistiche sull'andamento futuro del debito pubblico. Non si fa quindi ricorso a funzioni di perdita sociale, associate al default e dipendenti dalla relazione strategica tra debitore e creditori, come nella precedente letteratura. Le proprietà dinamiche degli equilibri sono utili per indagare la loro qualità e desiderabilità, dalla prospettiva della BCE.

Abstract

There have been many attempts at solving the problem of determining the "fundamental value" of the credit spread of a government bond. This is particularly important in the case of Eurozone, where the ECB intervention on the government bonds' market is allowed only if the "spread" paid by the sovereign issuer is higher than the one justified by "fundamentals". The complication in determining what is a fair level of the spread stems from the fact that public debt sustainability depends on many factors, among them the level of interest rates paid. This sort of circularity between debt sustainability and interest rate paid by the sovereign issuer is the major source of complexity. This paper highlights a possible solution inside a simplified framework resembling the peculiar institutional settings of the Eurozone: no possibility of money-financing, the famous Maastricht Treaty 3%-60% parameters, availability of financial assistance program subordinated to the acceptance of consolidation plans for public finances. We obtain the possibility of multiple equilibria for the credit spread, whose stability can be analyzed through a phase diagram. The dynamics of the model is derived from probabilistic assumptions about the public debt process. It does not depend on "loss" functions devised to model the strategic relationship between debtors and creditors, as in previous literature on public debt sustainability. Dynamic properties of equilibria can be used to gain insight on what does it mean "good" or "bad" equilibrium from the perspective of the ECB.

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Insegna International Financial Markets presso l'Università Cattaneo di Castellanza. Dal 1990 al 2000 è stato economista presso l'Ufficio Studi della Banca Commerciale Italiana (ora Intesa Sanpaolo), dove è stato responsabile della Financial Markets Research. Successivamente, ha svolto diversi incarichi nelle principali SGR italiane (Sanpaolo AM e Pioneer Investments), in Banca Patrimoni Sella e in UnipolSAI. Ha scritto articoli pubblicati su riviste internazionali. E' laureato in Università Bocconi (DES) e ha conseguito il MSc/MPhil in Economics presso la London School of Economics.

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