BUILDING COMPOSITE LEADING INDEXES IN A DYNAMIC FACTOR MODEL FRAMEWORK: A NEW PROPOSAL

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1 Introduction

Until the end of eighties, given the limited availability of data and computing tools and because of the generalized low efficiency of the estimation techniques, the academic macroeconometric models for forecasting were usually built on the basis of the principle of parsimony and involved only a handful of variables.

On the other side, practitioners, who need to provide in real time a complete picture of the state of the economy and give both policy makers and business men accurate information and forecasts, usually took into account in their analysis a large number of variables even in the absence of benchmark methodologies certified by academic research.

In the last twenty years the literature on macroeconometric forecasting tried to fill this gap between the activity of practitioners and the academic work.

One of the most significant steps of this process has been the growing academic interest in the composite leading indexes, previously used extensively for forecasting only within institutional frameworks.

Composite leading indexes provide the opportunity to exploit a much richer base of information than is conventionally used for time series analysis and forecasting, so they are considered a very useful tool for predicting future economic conditions. Since every economic phenomenon is affected by many different determinants, an aggregation of various indicators into composite indexes should show a stronger ability than a single indicator in capturing the current and future economic signals.

Moreover, in their simpler forms composite indexes are quite easy to construct and their dynamics are easy to interpret and to explain which are features much like to practitioners and policy makers.

Finally, composite indexes have attracted a considerable attention due to their capacity of summarizing and describing latent economic phenomena that affect a large number of macroeconomic series, like in primis the business cycle.

At first, the construction of composite indexes has developed mainly under non model-based approaches: since the pioneering work of Burns and Mitchell (1946), many institutions tried to measure (and lead) business cycles by aggregation, in the form of simple (weighted) averages, of a large set of cyclical (leading) variables.

Famous examples of this line of research are the indexes produced by the NBER business cycle dating committee\(^1\) in Europe, by the Conference Board\(^2\) in the US and by OECD, all aiming to find the historical cyclical turning points and predict the future ones.

It is evident the easiness of this approach that without an intensive use of technicalities is a very simple way to provide quickly interpretable economic indications. Anyway, the non model based indexes have been strongly criticized by Koopmans (1947) who defined them "measurement without theory", in that

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\(^1\) NBER examines the joint evolution of Industrial Production, employment, sales and real disposable income (Hall et al., 2003)

\(^2\) Previously by the Department of Commerce
stressing the statistical nature of this methodology that leaves room for economic theory only in the starting step of selection of variables candidate at the confluence in the final composite index.

This typically draws a kind of unsolved trade off between parsimony and coverage degree of the composite leading index. Larger is the set of input components and more information it does contain about the target variable, but without a theoretically founded criterion for aggregation its signal extraction capacity could rapidly deteriorate.

Many other main drawbacks (Emerson and Hendry, 1996; Marcellino 2006) affect the non model-based approach:

1) The adopted weighting schemes for the components aggregation are usually exogenously determined in the sense that they are not based on theoretical a-priori, do not refer to the intensity of comovements between component variables and the target and do not reflect any statistical criterion of optimality. All this stands at the origin of the frequent use of an “agnostic” strategy based on equal weights that seems optimal in the case of forecast pooling (Stock e Watson 2003).

2) It is not possible to evaluate the specific contribution of each single component to the dynamic behaviour of the final composite index.

3) An intensive use of preliminary data mining techniques is needed in order to minimize the noise into the final index.

4) One has to pre-select ex-ante as components only those variables characterized by the preferred lead; as an alternative, the series whose maximum leading capacity is placed in correspondence to horizons different from that preferred must be opportune shifted and re-aligned.

Different model-based methods have been developed, in more recent periods, in order to give the previous issues an answer and improve the techniques of exploiting many predictors for forecasting with leading indexes (for detailed survey see Stock and Watson 2006, henceforth SW06 and Marcellino 2006); among them the use of Dynamic Factor Models\(^3\) (henceforth DFMs) has assumed a particular relevance.

In his Ph.D. thesis Geweke (1977), moving from the observation of strong comovements among macroeconomic series\(^4\), introduced the Dynamic Factor representation, expressing each economic variable as the sum of a distributed lag of a small number of unoberved common\(^5\) factors plus an orthogonal idiosyncratic disturbance. In early applications to macro data Sargent and Sims (1977) and Sargent 1989) find empirical support to the view that a small number of common factors drive a large part of the observed variation in the economic aggregates; the perturbations affecting factors are just the common structural economic shocks the theoretical analysis and the policy makers are interested in, such as demand or supply shocks.

It clearly emerges that dynamic common factors could provide a “natural”

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\(^3\)Like forecast pooling and Bayesian model averaging.

\(^4\)As firstly stressed by Burns and Mitchell (1946)

\(^5\)Common to (near) all variables in the dataset
way of summarizing in a formal framework the informational content of large macroeconomic datasets and provide a sounder statistical basis for the construction of composite coincident and leading indexes. Their great advantage is to efficiently reduce the large dimensional problem of handling tons of variables to identify and estimate a very small number of components.

In a sequence of cornerstone papers, Stock and Watson (1989 - SW89, 1991, 1992) show how to obtain through the Kalman filter the maximum likelihood estimation of the parameters and the factors in a DFM cast into state space form and within this framework they rationalize and refine the U.S. Business cycle coincident composite index produced by the Conference Board.

Their index is obtained as the unique estimated factor of a low dimensional DFM allowing only for coincident variables. The corresponding n-period leading index may be obtained as the n-step ahead forecast of the coincident index based on a linear combination of past values of a group of pre-selected leading indicators.

Despite its exceptional innovativeness, the proposal by SW89 suffers three main drawbacks: (a) when n is very large (the most interesting case), maximizing the likelihood over so many parameters is too much consuming from the computational point of view, (b) the hypothesis that a unique common factor drives most of macroeconomic variables does not fit reality and (c) it is required an ex-ante classification of variables into coincident and leading ones.

Since SW89, a large body of literature has been developed on DFMs and forecasting: some lines of research have developed SW89 in an incremental way, whereas others have put forward proposals potentially alternative to it.

Stock and Watson (2002a, 2002b) address all issues (1) to (4) and show that with large datasets, including both coincident and leading (at all leads) variables, the consistent estimation of \( q > 1 \) dynamic factors can be based on static Principal Components Analysis (henceforth PCA), which is equivalent to solve a nonlinear least squares problem. Thus, they become evident the affinities between common dynamic factors and composite indexes in the sense that the estimated factors are just weighted averages of variables contained in the original dataset and that the weight system is an optimal one because minimises a quadratic loss function. Stock and Watson (2002a) in fact gives the estimated factors an interpretation in terms of “diffusion” indexes developed by NBER analysts to measure business cycles.

In this context, the generation of linear forecasts is directly obtained by using the \( h \)-step ahead formulation of the measurement equation of the DFM model.

Following an indirect two step procedure, past values of the previously estimated common factors can also be used within a dynamic linear equation in order to forecast a coincident index, some of its components or any other macroeconomic variable (Marcellino, Stock and Watson, 2003 and Banerjee, Marcellino and Masten, 2003 for the Euro area; Artis, Banerjee and Marcellino (2004) for UK).

To produce iterated \( h \)-step ahead forecasts, Favero, Marcellino and Neglia
(2005) and Bernanke, Boivin and Eliasz (2005) proposed an approach that models jointly as a VAR a block of pre-estimated factors (through static PCA) and a set of macroeconomic variables of interest. Such an approach, named Factor Augmented VAR (FAVAR), integrates factor methods into VAR analysis and provide a unified framework for structural VAR analysis using dynamic factors. Forni, Hallin, Lippi and Reichlin (henceforth FHLR; 2003b) and Giannone, Reichlin, Sala (2004) constrain the shocks onto the factors equation of VAR to have reduced dimension. Stock and Watson (2005) significantly refine the FAVAR approach taking into account the exclusion restrictions implied by the DFM.

FHLR propose two approaches alternative to that of SW, for the estimate of a DFM, anyhow based on the use of the Principal Components.

In the former (FHLR 2003a), the loss function to be minimised for the estimation depends on the inverse of the variance and covariance matrix of the idiosyncratic component.

Within a further line of research FHLR (2000, 2001, 2004) switch from static to dynamic PCA and apply this alternative methodology to the derivation of a composite coincident index for the Euro area. This method allows for a richer dynamic structure than static PCA, but it is based on two-sided filters so its use for forecasting requires trimming the data at the end of the sample. The way for a real time implementation of the dynamic PCA approach was showed by Altissimo et alii (2001b) and it is now adopted by CEPR in order to provide its composite coincident indicator for the Euro area (Eurocoin) which is the single leading factor estimated from a panel of nearly 1000 economic series. To construct n-step ahead predictions of the coincident index (Altissimo et alii, 2001a) one may project it n-step ahead on its current and past values and on simple averages of the common components of the leading variables contained in the dataset, endogenously selected on the basis of their lead relation with respect to the coincident index.

A one-sided version of the FHLR filter is used by Giannone, Reichlin and Sala (2004) to produce factor-based predictions of US GDP growth and inflation rate, aimed at miming the US Grenbook forecasts.

It is worth noting two main issues.

Curiously, the recent (PCA based) DFM literature, although constituting the physiological framework for the creation of composite Leading Indexes, has almost completely ignored this line of work, focusing instead on the insertion of the past values of previously estimated factors, as predictors in uniequational or multi-equational models (VARs) used for macroeconomic forecasting (for the static PCA approach Stock and Watson, 1999 and 2002b; Brisson, Campbell e Galbraith, 2003; Artis, Bancroft e Marcellino, 2004. For the dynamic PCA approach FHLR, 2000 and 2003 . For a comparison between static and weighted static PCA Bernanke and Boivin, 2003 and Boivin and Ng, 2003). Nevertheless, the evidence in favour of an outperforming forecasting ability of these factor

Stock and Watson (2006) use the expression “Weighted static PCA” to describe this approach.
based models with respect to more traditional approaches is mixed, mainly as a consequence of some low efficiency problem.

The key issue is the lack of construction of a synthetic composite leading index: of course, it is possible, once specified the forecasting equation (system of equations) for the target variable, to conceive a composite leading index as the non stochastic linear combination of the regressors. However, by doing this, aggregation weights (the estimated equation coefficients) end up depending on the particular specification of the model and are identical for any leading horizon; furthermore, as the factors used as predictors have not been economically identified, it becomes impossible to define the leading contribution of the main macroeconomic phenomena they describe, which surely damages the choices of policy makers (Think for example of the case of a Central Banker adopting an inflation targeting approach).

Only FHLR (2001, 2003a and 2003b) try to build a "true" composite Leading index, as the equal weight average of the common components of a set of leading indicators identified on the basis of their phase delay with respect to a composite coincident index estimated with dynamic PCA. Anyway, this two step procedure neither allows to determine the contribution of each factor to the dynamic of the leading index, nor to recover from the DFM estimate an optimal weight system for aggregation.

The previous issue gives origin to a second one: the consideration (SW, 2005b; Banerjee, Marcellino and Masten, 2003a) that the economic identification of the estimated factors (SW 2005 for the static PCA approach, Forni et alii, 2007 for the dynamic one) is not a crucial step for forecasting, seeing that what matters is simply that those factors help to improve the efficiency of forecasts. In some cases the identification problem is even bypassed, imposing the existence of a single factor, or solved on the basis of stylised facts, recalling that the stochastic component of most economic systems is most likely size 2, and that therefore it can be synthesised by two factors whose identification is known a-priori: one of them is representative of the real business cycle and the other of monetary phenomena.

In this paper we propose a self contained DFM based three step procedure for the construction of synthetic composite leading indexes with the aim of addressing all the issues that are still unsolved.

Our approach is oriented to the case in which the number of estimated factors is strictly higher than 1 and is suitable to forecasting any target economic phenomenon.

In particular, it allows to:

- Construct an entire set of leading indexes, each related to a different leading time, with no need to previously determine the lead relation between target variable and the single components, nor to eliminate phase misalignments;

- Endogenously generate the optimal set of weights for the aggregation of the components in correspondence to each leading horizon.
• Identify economically and formally the impact on the leading index of macroeconomic shocks of interest, as shocks to common factors.

The first step of the procedure schedules the estimate along the lines of SW (2005b) of a FAVAR model based on a large dataset including all variables that are supposed to influence the target phenomenon with no distinction between coincident and leading.

In the second step the structural shocks to the estimated dynamic factors are formally and economically identified on the basis of sign restrictions, extending to the FAVAR framework what is proposed by Canova and De Nicolò (2002), Uhlig (1999) and Rubio-Ramirez, Waggoner et Zha (2005; henceforth RWZ) for VAR models. By simulating the model over a \( k \) step-ahead simulation horizon it is possible to identify the impact of each shock on the target variable and draw, through the estimated FEVDs, a measure of the percentage contribution of each factor to the target variable variance corresponding to each \( i^{th} \) leading horizon \( (i = 1, \ldots, k) \).

These relative contributions define \( k \) sets of weights for aggregating factors (in a different way depending on the leading horizon) in order to get \( k \) composite leading indexes.

In this paper we apply the procedure to a particularly interesting case for practitioners and policy makers: the construction of a leading index for crude Nominal Oil Prices (henceforth NOPs).

Three factors are detected and estimated, that on the basis of sign restrictions are identified as representative of crude oil demand, of crude oil supply and of business cycle. From their weighted aggregation arises COPLI (Crude Oil Price Leading Index).

The remainder of the paper is organised as follows. In section 2 we present an overview of the lines of theoretical and empirical research on oil prices proposed in the academic literature. Our I/O theoretical model for the analysis of real wage, employment, participation and migration skilled-unskilled differentials. Section 4 is dedicated to estimation issues whereas empirical results and comments are presented in section 5. Section 6 concludes.

2 Theoretical and Empirical issues on Oil Prices: an overview

In order to effectively interpret the indications coming from the macroeconomic literature dedicated to analysing and forecasting the Oil Prices, it is doubtless useful to start from their time evolution analysis.

In accordance to a widespread common knowledge and following some recent contributions (Krishen, 2005; Mitchell, 2006) the evolution of both nominal and real\(^7\) crude oil prices is the result of the combination of 3 types of phenomena: (1) long term trends, (2) a short-medium run volatility corresponding to the

\(^7\) Normalised with respect to some measure of the world level of prices, often proxied by the US GDP deflator.
typical frequencies of business cycles and (3) a series of jumps, occurring una tanta but of remarkable width.

Looking at Oil Prices over a secular horizon it may be observed that the result of the combination of ingredients 1 to 3 has been different depending on the considered period. From the beginning of the 20th century until 1973 the oil market is not a spot one: seven major private-sector oil producer companies manage oil supply and exports and set prices in order to maximise their profits and match the world demand. A strong price stability in correspondence to relatively low levels has been the main consequence of this effective matching between supply and demand: markets were nearly cleared and the Oil Price profile in that period does not highlight any significant spike.

Starting from 1973 and until the first half of the ’80s the overall picture modifies due to the effect of two main factors: (1) the numerous shocks due to events of political and militar nature occurring in the ’70s, leading to great positive jumps in prices and (2) the beginning in 1982 of the oil market management by OPEC (the quotas regime: see Skeet, 1988) which aimed at sustaining the short run level of Prices through the rationing of effective supply, in spite of the existence of a high unused surplus of potential production capacity.

Nevertheless, apart from outliers, real Oil Prices show a tendency to reduction for a large extent of the ’70s and through all the ’80s which underlines the relative fragility and failure of quotas regimes and reveals how Prices are determined mainly by market forces, demand and supply shocks.

After 1985, with the abortion of the OPEC regime and because of the increased oil supply by new oil exporting countries, the market becomes more and more perfect, and the possibility of a binding rationing on the supply side seems a hardly credible threat. On the other side, world containment policies of oil demand (also through high energy taxes) contribute to contain in the short-medium run both the average level and the volatility of crude Oil Prices. Moreover, the increase of productivity that has occurred in almost all sectors of commodities seems to have triggered a very slow downward secular trend of their prices.

In the last years (after 2003), even in the context of a more efficient market, the economic explosion of some macro-areas of the planet, has fostered a strong oil demand, exhausting almost all the surplus of production capacity available in the short run. Consequently, both mean levels of crude Oil Prices and their fluctuations at business cycle frequencies have restarted to grow. In synthesis two stylised facts are apparent:

1. Oil Prices are determined by:

   - Oil supply shocks driven by one shot political events.

   It is worth noting that Oil Prices even exhibit strong volatility at the highest frequencies. It has origin mainly in arbitrage mechanisms and in speculative behaviours occurring on financial markets. Given the macroeconomic approach of this paper, we will not deal with such feature.

   In accordance to the well known Prebish-Singer hypothesis on which, moreover, the empirical evidence is mixed.
• "Market" oil supply and demand shocks occurring at business cycle frequencies

• The long run upward trend of productivity in the commodity sectors.

2. During the last 40 years the role of triggering determinant of Oil Prices has gradually shifted from oil supply to oil demand.

In spite of the enormous importance that oil markets have in global economy and the large academic debate on the impact that oil shocks have on economic systems (see Blanchard and Gali, 2007 for a survey), the body of macroeconomic literature, both theoretical and empirical, focused on modelling and forecasting the behaviour of Oil Prices has a limited size.

In most cases in fact, real oil prices have been considered as an exogenous variable, or, following a microeconomic approach, were supposed being a positive and complex function of marginal costs of production (extraction and refinement costs). When endogenously modelled, however, Oil Prices have been in many cases described through simple single-equation reduced form models, without allowing for any economic structural content. Moreover, it is worth noting that in some contributions, although relevant, the specific features of the oil markets were not analysed "in isolation", but assimilated to those of the widest family of commodities, in the light of a (debated) significant and positive correlation between oil and non oil commodities (Pindyck e Rotemberg 1990).

Works dedicated to the study of oil prices have as their main goal that of surveying the stylised fact (1) and to disentangle the oil price movements into a cyclical component, mainly driven from demand pull factors, and a long term trended movement strongly affected from structural supply side dynamics. This sort of dichotomy is re-proposed also in forecasting models, with the use of more demand oriented models for short term forecasts and models which stress structural determinants if the goal is to produce long term forecasts. In contrast with the stylised fact (2), the focus was first mainly placed on the analysis of the role of the demand factors, usually carried out in the sphere of partial equilibrium approaches, to reach only more recently to extend organically also to the role of supply.

Considering the monopolistic structure of the oil market, controlled by the OPEC cartel up to the mid '80s, a number of models (Houthakker, 1976; Pindyck, 1978; Lowinger et alii, 1983; Lowinger and Ram, 1984) have been developed in order to explain the OPEC's decisions with respect to pricing and production behaviour. In such a literature OPEC is perceived as a monopolist that sets the price of his product (with reference to the demand for it) mainly on the basis of the realisation value of oil as determined by refiners in terms of the value of oil-derived products.

With the decline of the OPEC regime, another line of research (Chu and Morrison, 1986; Dornbusch, 1985; Gilbert, 1989), on the basis of partial equi-

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10 There exists also a literature directly inspired by financial economic theory and based on the market efficiency hypothesis. We are not interested in this family of contributions. For a short survey see Longo et alii (2007)

11 Which according to De Santis (2000) remains the main cause of oil price fluctuations
librium models, has identified in the business cycle of industrialised countries and in the Dollar real exchange rate the main demand-side determinants that foster the consumption of oil and non oil raw materials, influencing their prices, mainly in the short run. A positive business cycle phase stimulates aggregate demand, energy demand and prices; on the other hand, a U.S. Dollar depreciation reduces the profit margins for the Oil exporter countries which try to raise prices in dollar terms in order to recuperate losses coming from currency appreciation.

This partial vision turns out to be inappropriate by the end of the '80s: the stagnating trend of Oil Prices is not correctly explicable on the basis of the joint contribution of only business cycle variables and exchange rates, and forecasts based of them result systematically upward biased\footnote{From the econometric point of view, only the demand side models suffer from both misspecification and simultaneity bias}.

The '90s, therefore, highlight the start of a more modern and complete approach to the market of raw materials (oil and non oil): the aim becomes that of distinguishing the determinants that in the short run act on prices with temporary effects from those generating a more persistent impact in the long run, quantifying in this way the relative contribution of demand and supply shocks.

In a cornerstone paper, focused mainly on non oil commodities, but extendible (in the authors view) also to the oil market, Borensztein and Reinhart (1994) complete the reconstruction of the demand side determinants allowing for the role of the newly industrialized Eastern European countries. However, their innovative contribution consists of the inclusion in what they call "full structural model" of an indicator which represents the commodity supply, although measured in an extremely approximate way, as sum of the industrial countries' imports of primary commodities.

Two strong indications emerge from the paper:

- The factor which approximates the supply, in accordance with the theoretical indications, seems to affect the real commodity prices in a negative manner.

- In terms of its forecasting performance the model outperforms some naive models (RW model) but only over medium-long term horizons (more than 5 quarters)

The existence of a dialectic between short term cyclical demand factors and medium term trend generating supply factors is traceable back to a large part of more recent literature dedicated to prices of raw materials and in particular to oil prices.

Accentuation is anyway different. Emphasis on long run phenomena acting mainly on the supply side is found in Reinhart e Wickam (1994), where productivity growth of the commodity sector generates a downward trended price over a secular horizon and later in Cashin et alii (2000, 2001) which find that most commodity prices are affected by long-lasting shocks and that in the very long
run productivity growth and institutional structural changes in supply policies play a significant role.

Elsewhere (Lalonde et alii, 2003), it is underlined that, taking into account the structural breaks that characterise supply conditions, the analysis and forecast of real Oil Prices benefit especially of an adequate modelling of the growing role that over time has assumed the cyclic component. In such case, even a simple uniequational dynamic model is able to forecast the evolution of prices in a way that outperforms wide-spread competitors such as RW, AR(1) and VAR models.

The interaction between demand and supply is also referred to by Krichene (2005) which specifies a SEM for world crude oil and natural gas markets (with both quantity and price variables) with the aim of assessing the role played by monetary shocks in the commodity markets. By estimating the reduced form (VAR model) of the SEM a significant evidence is found of an inverse relation between oil prices on one side and U.S. Nominal Effective Exchange Rate (NEER) and U.S. Interest Rate13 on the other; moreover the negative effects of NEER on prices seem to have greater size over the periods when interest rates and exchange rates exhibit a higher volatility. Demand factors like world oil demand and supply factors like the OECD oil stocks and the OPEC productive capacity (Kaufmann, 1995), or the OPEC quotas (Kaufmann, 2004; Dees et alii, 2007) are jointly included in a single equation model on real import oil prices and seem to be helpful in order to improve its in-sample static and dynamic forecasting performance. Ye et alii (2005) show that a model for WTI spot prices including a relative industrial oil stock measure (deviation of oil stocks from their normal level) produces better forecasts than simple autoregressive models.

Kilian (2007), using a newly developed measure of global real economic activity, proposes a structural decomposition of the real price of crude oil in four components: oil supply shocks driven by political events in OPEC countries, other oil supply shocks, aggregate shocks to the demand for industrial commodities and demand shocks that are specific to the crude oil market. The analysis suggests that shocks to the production of crude oil are less important in understanding changes in the real price of oil than shocks to aggregate demand; politically driven crude oil production disruptions do not systematically increase the real price of oil whereas other oil production disruptions cause a transitory increase in the real price of oil within the first year.

Longo et alii (2007) builds a set of different models for crude oil prices and chooses as regressors some measures of world oil consumption and production, government and industrial oil stocks; he makes a comparison among structural models and pure time series specifications but he does not find a clearly over-performing specification over all forecasting horizons, data frequencies and historical periods.

An attempt to conciliate the analysis of short and long run is found in

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13In the Krichene's view a low U.S. interest rate usually implies a world-wide low Interest Rate that stimulates Aggregate Demand and therefore energy demand and prices.
Hubbard (1986) which formulates a “two-price” model in which there is the coexistence of both short term spot prices and long term “contract” prices which diverge only in presence of demand or supply shocks. Temporary shocks, even supply shocks, impact on both prices and, the pass-through from spot prices to contract prices can make the effects of the shock, at least on these last ones, persistent if the contracts adjust gradually and there are no backward looking expectations.

A parallel stream of research (Bui and Pippenger, 1990; Cuddington and Liang, 1997) underlines that volatility of real primary commodity prices and in particular of petroleum prices is similar to that which affects exchange rates and usually grows in historical periods characterised by a greater monetary/currency flexibility. On the basis of GARCH models Cuddington and Liang (2000) infer in favour of a stabilising role of global oil markets following the introduction of the Euro.

Adopting a class of econometric equations suggested by Pindyck, Bernard et alii (2004) model the long run behaviour of energy prices as a mean reverting process with continuous and random changes in level and trend: however, differently from coal and natural gas prices, crude oil prices do not reveal any changing regime and they seem to be well modelled and forecasted by a not time varying parameter model.

Summarising:
1. Modelling and forecasting Oil Prices is definitely a complex operation which requires the adoption of an eclectic and flexible approach and especially the ability to manage large and heterogeneous datasets which include demand, price, supply and "context" variables.
2. Some stylised remarks are anyway possible:
   - There exist economic phenomena and variables fostering the oil demand that have a short-medium run positive impact on Oil Prices: we refer to country size variables (like the GDPs), variables which measure the economic cycle (Industrial production and more), interest and exchange rates. Moreover, specific indicators measuring oil demand and oil consumption should not be forgotten.
   - There exist economic variables describing phenomena acting on the oil supply side, which produce a mid-long run reducing effect on Oil Prices: these are the indicators of oil production, oil supply, oil stocks and oil reserves, but also variables measuring the productivity of the oil sector.

Given these considerations, the strategy we use in this paper to model and forecast Oil Prices seems to be highly profitable: DFMs in fact enable the efficient and nimble management of large heterogeneous datasets (point 1) and their identification, based on sign restrictions, enables taking into consideration all the issues summarized at point 2.
3 Dynamic Factor Models: specification, estimation and simulation

Dynamic Factor Models (DFMs) have been developed as a powerful tool for exploiting the information contained in large datasets and summarizing the covariances among the variables contained therein. DFMs allow to describe the behaviour of each series as the sum of two components: the dynamics of a reduced number of common factors and an idiosyncratic shock. Let us collect the $n$ variables of the dataset in the vector $X_t$ and $q$ common factors in vector $f_t$

The dynamic form of a DFM may be expressed as follows (SW, 2005):

$$X_t = \Lambda(L) f_t + D(L) X_{t-1} + \nu_t$$

where $n$ (usually large) is the number of variables in the model, $q$ the number of dynamic, primitive factors, $D(L)$ is a diagonal matrix lag polynomial $D(L) = \text{diag}(\delta_1(L), ..., \delta_n(L))$ and $\Lambda(L)$ has degree $p - 1$.

$E(f_t) = 0, E(\nu_t) = 0, E(f_t \nu_r) = [0]$  

Common factors ($f_t$) and idiosyncratic shocks are uncorrelated at all leads and lags.

Chamberlain and Rothschild (1983) make a distinction between exact and approximate DFMs; in the former case $E(\nu_t \nu_{j+r}) = 0, \forall i \neq j$, in the latter there exists some contemporaneous correlation.

Let us define a vector containing the so-called static factors:

$$F_t = [ f_{t-1}^1, f_{t-1}^2, ..., f_{t-p+1}^1 ]$$

The static form corresponding to system [1] is as follows:

$$X_t = \Lambda \Phi(L) F_{t-1} + G \eta_t$$

- $r = (p \times q)$ is the number of static factors ($F_t$).
- $\Phi(L)$ consists of the coefficients of $\Gamma(L)$ and zeros
- If order of $\Gamma(L)$ not higher than $p$, then $\Phi(L) = \Phi$
- If $p = 1$, static factors coincide with dynamic factors.

The VAR form of a DFM (FAVAR model; Bernanke, Boivin and Eliasz, 2005) might be obtained by substituting equation 2 of system [2] into equation 1:
The (1,1) block of $\Sigma_e$ contains the variance and covariance matrix of the static factors which is a function of its dynamic counterpart $\Sigma_\eta$; matrix $G$ relates dynamic and static factor innovations. Notice that:

- the $\varepsilon_{x,t}$ have factor structure
- the $\varepsilon_{F,t}$ have factor structure without idiosyncratic noise
- $\text{rank}(G) = \text{rank}(G_\eta G') = q$
- $G_\eta G'$ is positive semidefinite
- in the SW05 version\textsuperscript{14} $\Sigma_\eta = I$.

Inverting the system [3] and focusing on $X_t$ yields its MA representation in terms of current and lagged orthogonal innovations $\eta_t$ to the dynamic factors:

$$X_t = B(L)\eta_t + u_t$$

where:

- $B(L) = [I - D(L)L]^{-1} \Lambda [I - \Phi(L)L]^{-1} G$ and $u_t = [I - D(L)L]^{-1} \nu_t$
- impact multipliers: $B_0 = \Lambda G$
- long run multipliers: $B(1) = [I - D(1)]^{-1} \Lambda [I - \Phi(1)]^{-1} G$

Estimation may be obtained following a three step approach\textsuperscript{15} (SW05):

**Step 1:** Given the number of dynamic factors $q$, get $\widehat{P}, \widehat{\Lambda}, \widehat{D}(L)$, by solving iteratively the following minimization problem:

$$\min_{F_0,F_1,\ldots,F_t,\Lambda,D(L)} T^{-1} \sum_t [I - D(L)L] X_t - \Lambda F_t [I - D(L)L] X_t - \Lambda F_t$$

Solution requires:

\textsuperscript{14}The $\eta_i$ shocks are formally identified using an arbitrary statistical normalization even though they are not economically identified along the lines of some theoretical economic model.

\textsuperscript{15}For simplicity let us assume that we are in the condition in which the dynamics of $\Lambda(L)$ is no higher than $p$ (the loadings have lags which do not exceed the dynamics of dynamic factors.)
• step 1a: $F_t$ can be computed by applying static PCA to $	ilde{X}_t = [I_n - D(L)L]X_t$

• step 1b: regress $X_{it}$ on $F_t$ and on $X_{it-1}, \ldots, X_{it-m}$ to get estimate of $\delta_i(L)$ and $\Lambda$

Each step of this procedure reduces (does not increase) the sum of squares in (6) and the procedure can be iterated to convergence.

• step 1c: estimate the number of static factors $r$ using Bai and Ng (2002) IC criteria.

**Step 2**: get $\Phi(L)$, by auxiliary regressions (Fit a finite order VAR to $\tilde{F}_t$ with OLS)

**Step 3**: Let us consider the simplest case when $\Phi(L) = \Phi$ and $D(L) = D$.

The VMA representation of the FAVAR becomes:

$$X_t = (I - DL)^{-1}A(I - \Phi L)^{-1} \eta_t + v_t$$

$$B_t = A_t G$$

$$A_0 = A, A_i = DA_{i-1} + \Phi^i$$

With $G$ in hand we can obtain the IRFs and FEVDs for structural common shocks.

SW05 exploit the factor structure of $\varepsilon_{it}$ in order to get estimate of $\hat{G}$ and the space spanned by the dynamic factor innovations $\tilde{\eta}_t$, and recover the dynamic factors.

Let us normalise $\eta_t$ to have identity matrix; then we can write:

$$\Sigma_{\varepsilon_{it}} = E(A(L)G\eta_t \eta_t'G' A(L)) + \Sigma_v$$

and taking trace

$$\text{trace}(\Sigma_{\varepsilon_{it}}) = \text{trace} \left[ G \left( \sum_{i=0}^{\infty} A_i' A_i \right) G' \right] + \text{trace}(\Sigma_v)$$

therefore we are able to estimate $G$ to max trace $R^2$, by computing $G$ as the $q$ eigenvectors associated to the highest $q$ eigenvalues of $\sum_{i=0}^{\infty} A_i' A_i$. $G$ is then normalised to generate orthonormal disturbances via the relation $\varepsilon_{Ft} = G \hat{\eta}_t$.

The number of dynamic factors $q$ is estimated by applying the Bai-Ng (2002) procedure to the sample covariance matrix of $\varepsilon_{it}$, yielding an estimator $\hat{q}$. It is worth to note that this procedure finds the estimates of the innovations to the dynamic factors $\eta_t$ on the basis of an arbitrary statistical normalization and not a theoretical structural economic model; in other words the impulse responses and variance decompositions delivered by the VMA representation of the DFM can be thought of as the factor version of impulse responses and

\footnote{Notice that the VAR residuals variance and covariance matrix will have rank equal to $q$.}
variance decompositions with respect to Cholesky factorizations of conventional VAR innovations. The dynamic factor structural shocks $\zeta_t$, that is the orthogonal shocks admitting an economic interpretation, are assumed to be linearly related to the reduced form dynamic factor innovations by:

$$\zeta_t = H\eta_t$$

where $H$ is an invertible $q \times q$ matrix and $E(\zeta_t, \zeta_t') = I$ so that $H\Sigma_r H' = I$.

In order to achieve the really structural dynamic factor shocks $\zeta_t$, Stock and Watson (2005) illustrate a set of different strategies, all based on zero restrictions on dynamic multipliers, as they have been proposed in the structural VAR literature. Christiano, Eichenbaum and Evans (1999) adopt a recursive identification scheme based on restrictions on the impact multipliers and inspire the Bernanke, Boivin and Eliasz (2005) FAVAR proposal, whereas Blanchard and Quah (1989) impose long run restrictions first used in FAVAR models by Giannone, Reichlin, and Sala (2002).

Anyway, exclusion restrictions have been strongly criticized in the literature: Faust and Leeper (1997) show that small sample bias and measurement errors may induce substantial distortions in the estimations when using long run zero restrictions. On the other side, short run restrictions may be too stringent and misleading; in many cases they are introduced not due to theoretical foundations but they are arbitrary imposed to respect order and rank conditions for identification. Moreover, Peersman (2004) shows that a large number of impulse responses based on zero restrictions are located in the tails of the distributions of all possible impulse responses.

In order to avoid technical problems of this sort in this paper we follow an identification strategy based on sign restrictions (Faust, 1998; Uhlig, 1999; Canova e De Nicolò, 2002): different dynamic factor shocks are identified according to the direction of their impact on the variables in the system. Canova and Paustian (2007) show the many advantages of this strategy compared to an alternative one based on classical or Bayesian structural estimation. Firstly it is not necessary to make strong assumptions on the true DGP of the data, like in classical estimation; on the other side one can avoid the large computational costs and the difficulties of interpretation of misspecified estimates not infrequent in the structural Bayesian approach.

Ramirez, Waggoner and Zha (2005; RWZ) provide a multi-step algorithm for SVAR identification based on sign restrictions directly imposed on impulse responses; its extension to a FAVAR framework is quite straightforward.

**Step 1.**

- **Pre-condition:** following RWZ the procedure should start with a posterior draw of the model parameters in any exactly identified SVAR. In the case of our FAVAR we satisfy the requirement of the exact identification of the model, but we are not able to simulate the distribution of its estimated parameters; so we start from the "classical" FAVAR point estimation described in the previous subsection.
- **Generate** $\widehat{B}_0 = \widehat{\Lambda} \widehat{G}$
Step 2. Draw an independent standard normal $q \times q$ matrix and set $Z = QR$, where $QR$ is the output of a QR decomposition, the diagonal of $R$ is normalized to be positive and $Q$ is an orthogonal (rotation) matrix.

Step 3. Generate $Q^{-1}B_0 = Q^{-1} \Lambda \Gamma$; keep IRFs if they satisfy the sign restrictions and discard them if not.

Repeat step 2 and 3 $n$ times and calculate first and second order moments of the functions of interest.

4 The empirically estimated model

4.1 Data and sources

In this sub-section we describe the construction of the variables used in the empirical work, their statistical properties and the basic specification of the DFM model.

The dataset contains 151 series covering the whole set of economic variables affecting the crude oil market: a nominal crude Oil-Brent price series (US Dollars per Barrel), the real GDP and the Industrial Production series of the largest world economies (21 countries; more than 80% of the global World GDP), the crude oil demand/consumption indicators for the main oil importers, and the oil production series (more than 90% of the world total production) for the main exporters. For the exporter countries we include in the dataset also the series of oil reserves and for a subset of them we add the oil stocks series. Moreover, following the consensus view expressed by the academic literature on oil prices we complete the dataset with the Real Exchange Rate series for EU-12 and US and four interest rate series: the US Treasury Bills and the Euro-market 3-Month Interest Rates for the short run, the US Treasury Bond (10 Years) and the Euro-market 10-Year Government Bonds for the long run. Finally we add to the dataset the series of lagged values of Nominal Oil Prices (NOP). The source of data is Datastream, except for Oil Reserves and Stocks for which the source is the US Department of Energy.

All series are seasonally adjusted and have been filtered with TRAM-OSEATS in order to detect and remove the largest outliers. In the estimation we used their logarithmic transformation (save for the interest rates) and since

17 And FEVDs.
18 Argentina, Australia, Brasil, Canada, China, France, Germany, Japan, India, Indonesia, Italy, Korea, Mexico, Russian Federation, Spain, South Africa, Thailand, Taiwan, Turkey, UK and US
19 UK, US, Germany, China, Canada, France, Italy, India, Japan, Russian Federation, the area of Pacific Basin, other OECD countries and other countries of the Eastern Europe.
20 Algeria, Argentina, Angola, Australia, Brazil, Columbia, China, Canada, Ecuador, Egypt, Gabon, Iran, Indonesia, India, Iraq, Kuwait, Libya, Mexico, Malaysia, Nigeria, Norway, Oman, Qatar, Russia, Saudi Arabia, Syria, United Arab Emirates, United Kingdom, United States, Venezuela.
21 to 6 periods.
they result being I(1) we fit the DFM to their first differences. Finally we
standardized them to remove some scale asymmetries.

Data are monthly sampled and the time horizon we consider comes from
1987M1 to 2006M12;

4.2 Specification issues

Before describing the guidelines of our empirical experiment a premise seems
to be necessary: all the steps of the analysis and also the results they have
produced have to be considered as strongly preliminar and it is evident that
various improvements and refinements are required to enable us to consider them
as robust, satisfactory and definitive. Nevertheless we think that, although being
basic and stylized, the empirical evidence contained in this paper clearly suggests
that a fine combination of DFMs and Forecast Error Variance Decompositions
is potentially a powerful framework to produce Composite Leading Indexes with
very appealing forecasting performances.

First step of the empirical experiment has been the estimation of a FAVAR
model exploiting the informations contained in our large dataset along the lines
proposed in the previous section. We adopted a quite simple DFM specification:
we supposed that the order of dynamics of factors in the measurement equation
(of the dynamic form) of the DFM is 0; it becomes 1 when we take into account
that the idiosyncratic shocks are supposed to be generated from a stationary
AR(1) process. As for the state equation, we modeled the evolution of dynamic
factors as driven by a VAR(1) model. Finally we suppose that there exist
three dynamic factors22. After having estimated the DFM with static PCA, the
second step is focused on the (formal and economic) identification of the three
estimated dynamic factors as representing respectively the Oil Demand, the Oil
Supply and the business cycle dynamics hidden in the common components of
all series.

Due to computational problems we have neither constrained all the (151 x 3)
impact multipliers of our FAVAR, nor followed all the theoretical suggestions
on them, but we have built a subset of restrictions that enable us to distinguish
the instantaneous effects of the three factors. In detail, after having collected
dynamic factors in a (3 x 1) vector \( f_t = [Fac1 \ Fac2 \ Fac3]' \)
we suppose that:

- \( \frac{\partial \text{OIL PRICE}}{\partial \text{OIL DEM}} > 0, \frac{\partial \text{OIL RESERVES}}{\partial \text{OIL DEM}} < 0 \) with \( j = 1, \ldots, 30 \)
- \( \frac{\partial \text{OIL PRICE}}{\partial \text{OIL SUPPLY}} < 0, \frac{\partial \text{OIL RESERVES}}{\partial \text{OIL SUPPLY}} > 0 \) with \( j = 1, \ldots, 30 \)
- \( \frac{\partial \text{OIL PRICE}}{\partial \text{BusCyc}} > 0, \frac{\partial \text{IND PROD}}{\partial \text{BusCyc}} > 0 \) with \( i = 1, \ldots, 21 \)

This way we restricted 84 impact multipliers23.

22Running the IC criteria by Bai-Ng (2002) we have found some support to this assumption.
23Various set of restrictions alternative to the one reported in the text have been specified;
however results appear to be quite robust with respect to this choice.
To check the restrictions and simulate the FAVAR model in order to derive Impulse Response Functions (henceforth IRFs) and Forecast Error Variance Decompositions (FEVDs) we have adapted the multi step procedure proposed for VAR model by Rubio-Ramírez et alii (2005) that we presented in the previous section.

After having run 1000 replications of the three steps algorithm, we look at the FEVDs of the oil price variable which give a measure of the percentage contribution of each one of the three factors/components to the variance of the target variable for each one of the $k (=20)$ considered simulation horizons. These percentage contributions are the weights defining $k$ weighting schemes used for the aggregation of the orthogonal factors in order to generate a set of horizon-specific composite leading indexes of the oil prices\footnote{We used as target variable to be leaded both the nominal oil prices and also the real ones. Since the model revealed better leading performances in the former case, we reported only the results for nominal prices.}.

5 Estimation results and comments

It is worth to stress that the empirical results presented in this section are strongly preliminary and should not be taken as definitive; in particular we think reasonable to adopt an agnostic attitude to them. On one side too much optimism is inadvisable: results are still too much sensitive to the main model specification choices, as regards the number of estimated factors, the order of dynamics characterizing both measurement and state equation, the sample size, number and form of constraints on impact multipliers and more. Moreover, conditionally on these choices, we detect a high degree of uncertainty around simulated median values of IRFs and FEVDs, which does not seem to reduce even raising the number of iterations. On the other side some appealing properties of the simulated coincident and leading indicators are encouraging just because obtained through a rather crude exercise: there exist large margins of improvement, exploitable through a higher quality of data included in the dataset, a refinement of factor identification procedures in order to manage a higher number of restrictions and the adoption of a more sophisticated specification for the starting FAVAR model. We are actually testing these lines of research with positive results.

Figure 1 plots NOP series and the composite coincident ($k = 0$) index (CCI) in levels; FEVDs we have obtained by simulation suggest to give the three factors $[OIL\_DEm \ OIL\_SUPPLY \ BusCycI]$ the following weights:

$[0.21 \ 0.5 \ 0.28]$. 

The visual inspection reveals a quite noticeable behaviour of the composite coincident index which captures in real time almost all the relevant (medium run) turning points of the target variable: the end of the price decline at the beginning of the 90s, the negative phase in 1993 and 1994 and the subsequent growth cycle till the third quarter of 1997, a renewed prolonged decline and
then the price recovery started in 2001. The performance of the index at higher frequencies is less convincing and month to month peaks are not always well captured: the inclusion in the database of some quarterly series (GDP and Oil reserves) could be the origin of a modest ability of the system to correctly interpret phenomena occurring at a monthly frequency. Anyway the correlation between target variable and coincident index is strongly positive (0.75) and we think that the index behaviour is quite satisfying.

NOP and CCI are characterized by similar statistical properties: both exhibit a high partial autocorrelation of order one, both are I(1) and finally we are not able to reject the hypothesis of equality of their means and variances.\textsuperscript{25}

Let us move to analyse the performance of the \((k - 1)\) composite leading indexes we have built, one for each \(i^{th}\) leading horizon \((i = 1, \ldots, k)\), taking as a benchmark the three-months leading index \((k = 3)\) that we plot in Figure 2 with the target NOP. Many of the comments which we have done before are confirmed: it is evident a valid leading performance of the composite index with respect to the macro-turning points of NOP, whereas its sensitivity to short-term fluctuations appears to be weaker.

Both arguments are softly strengthened by plotting the moving averages counterparts of the series (figure 3) and also by comparing the correlation coefficient\textsuperscript{26} between them (0.82) with the one calculated on level series (0.72).

In order to provide a summarizing sketch of the leading performance at different horizons we report in Appendix the series of correlations between target NOP series and all the \(k\) Leading Composite Indexes: as usual, the quality of

\textsuperscript{25} A puzzling and quite warning evidence is that NOP and CCI are not cointegrated on the basis of the famous Johansen Trace test.

\textsuperscript{26}Correlation coefficients are obtained shifting the target series three months forward.

Figure 2:

![Nominal Oil Prices and Composite 3-months Leading Index](image)

Figure 3:

![Nominal Oil Prices and Composite 3-month Leading Index (Moving Averages)](image)
leading performance deteriorates as $k$ grows.

Some interesting indication emerges from the evolution of FEVDs with respect to the simulation horizon (Figure 5). When we consider the instantaneous effects of shocks NOP variance seems to be affected mainly from perturbations in the Oil Supply and to a lower extent from Business Cycle dynamics. The latter become the main contributors to explain NOP evolution in the medium run (two to seven steps ahead), when also the role of Oil Demand acquires greater importance. In the long run (eight to twenty steps ahead) the relative contributions of Oil demand and Supply are fully reversed with respect to the short run and Oil Demand shocks are protagonist. Differently from the common view of some literature, our very preliminar results could suggest that Oil Demand has a growing trended behaviour which is dominated from structural economic phenomena (the economic growth of China, India) acting over longer horizons, whereas Oil supply mainly influences the cyclical movements of the NOP\(^{27}\).

\(^{27}\)In order to balance the model the inclusion of a variable measuring the long run oil supply evolution (like productivity in the oil sector) should be recommended.
6 Concluding remarks

One of the most problematic aspects in the work of policy makers and practitioners is having efficient forecasting tools that combine two seemingly incompatible features: parsimony and ease of use on one side, completeness of the information set underlying the forecasts on the other.

The large and heterogeneous body of recent econometric literature provides different answers to the two needs: Dynamic Factor Models (DFMs) are born to optimally exploit the information coming from large datasets and to bring complex correlation structures to a few estimated common factors; composite leading indexes represent an immediate and flexible tool to anticipate the future evolution of a phenomenon and especially its turning points.

This paper fills the gap and proposes a multi-step procedure for building composite leading indexes within a DFM framework. Once selected the target economic variable we intend to forecast, first step consists in specifying and estimating a DFM in FAVAR form (Bernanke, Boivin and Eliasz, 2005; Stock and Watson, 2005) which draws information from a large dataset containing all the variables that are correlated to the target, and in primis the leading ones. The estimated common factors summarize all the relevant (and not idiosyncratic) dynamics on target behaviour. It then proceeds to the formal and economic identification of the orthogonal shocks affecting common factors and to simulate the structural form of the FAVAR: to this end we adopt sign restrictions on the impact multipliers. The Forecast Error Variance Decompositions obtained over a \((k + 1)\) steps-ahead simulation horizon are the key objects to our goal. Since they describe the percentage contribution of a shock (factor) occurred \(j\) periods before to the evolution of the target phenomenon, they define \(k + 1\) sets of optimal weights for aggregating factors, in a different way depending on the considered horizon, in order to get composite coincident indexes \((j = 0)\) or leading indexes \((j = 1, \ldots, k)\).

In this paper we run a very preliminary empirical exercise aimed at forecasting crude nominal oil prices. The results seem to be encouraging and support the validity of the guidelines of our proposal: we generate a wide range of horizon-specific leading indexes with appreciable forecasting performances. In addition, it appears that the behaviour of prices is significantly related to the business cycle but is mainly driven by the gap between oil demand and oil supply with variable weights of either depending on the horizon taken into consideration.

We think, however, it is necessary to further work to improve the procedure along different directions: refining and enlarging the dataset, improving the identification procedure in order to overcome the current limitations that prevent imposing a large enough number of sign constraints, evaluation of the potential contribution of the idiosyncratic component, adoption of more sophisticated criteria for assessing the forecasting performance of the leading indexes.
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Appendix

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