NEW TRADING METHODOLOGY FOR FINANCIAL TIME SERIES

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1. Introduction

A fundamental assumption in modern finance is the Efficient Market Hypothesis (EMH) [1] which states that in a well-functioning and informed capital market, the entire history of information regarding an asset is reflected in its price and that the market responds instantaneously to new information. Therefore EMH implies that attempts to use past price data to predict future values are doomed to failure, i.e. no profitable information about future movements can be obtained by studying the past prices series. The earliest form [2] assumed

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that market movements are described by stochastic process, i.e. random walk theory. However, this is actually considered as not true [3].

Recently, with the development of complex systems theory, there has been an increasing interest in the application of concepts and methods developed in non-linear mathematics and physics to problems in economics and finance under the rubric of ‘econophysics’ [4-5]. This new field of research has questioned several of the basic assumptions of standard finance theory, between them:

- Empirical evidence strongly suggest that the probability distribution functions found in financial time series exhibits a fat – tailed distribution which is in disagreement with the Gaussian distribution and the random walk model [5-6].
- Even though the autocorrelation function is essentially zero for all time lags bigger than zero, and, therefore consistent with standard finance theory, there are higher order temporal correlations that survive over long time periods [4].
- The EMH does not hold in financial markets and there are trading opportunities but the gain is too small when compared with the transaction costs to take full advantage [7]. However, net gain does not necessarily imply that EMH is incorrect [3].

In this work, we have applied state space reconstruction techniques to estimate state space volume and its variation. These values have allowed us to define a trading methodology by considering a sort of acceleration in a high-dimensional state space system as a kind of momentum indicator similar to those used in financial technical analysis [8-9]. Our interest was to develop a general trading strategy to determine and quantify the amount of predictability in these time series. This trading methodology has been applied to high-frequency currency exchange time series data from the HFDF96 data set provided by Olsen & Associates [10]. The time series studied are the exchange rates between the US Dollar and 18 other foreign currencies from the Euro zone; i.e. Belgium Franc (BEF), Finnish Markka (FIM), German Mark (DEM), Spanish peseta (ESP), French Frank (FRF), Italian Lira (ITL), Dutch Guilder (NLG), and finally ECU (XEU); and from outside the Euro zone: Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Frank (CHF), Danish Krone (DKK), British Pound (GBP), Malaysian Ringgit (MYR), Japanese Yen (JPY), Swedish Krona (SEK), Singapore Dollar (SGD), and South African Rand (ZAR).

2. Methods and approach

In the last few years, non-linear time series analysis has been rapidly applied in all the scientific fields [11-12]. It comprises a set of techniques for the analysis, manipulation, and understanding of aperiodic signals relying on the hypothesis of deterministic chaos [11]. This means that the signal reflects the complex dynamics of a low dimensional deterministic system in the long term limit, i.e. when $t \to \infty$. However, these signals represent a very limited class of real time series and therefore, there is a considerable interest to explore how far these concepts can be applied more generally to aperiodic signals with non-deterministic origin and strong non-stationarity.

Financial time series represent such as systems, they are normally analysed as using the theory of Brownian motion [13] or fractional Brownian motion [14], and they are non-stationary time series [15]. Therefore the typical tools developed for analysing chaotic time series present serious limitations when applied to these time series.

2.1. State space reconstruction and divergence calculation

In order to analyse non-linear time series we have used the theory of embedding. The theory of embedding is a way to move from a temporal time series of measurements, $s(t)=h[\mathbf{x}(t)]$, which is related to the state variables, $\mathbf{x}(t)$, by a unknown function, h, to a state space similar -in a topological sense- to that of the underlying dynamical system we are interested in analysing. Techniques of state space reconstruction were introduced in [16-17], who showed it is possible to address this problem using time delay embedding vectors of the original measurements, i.e. $\{s(t),\ s(t-\Delta t),\ s(t-2\Delta t),\ldots,\ s(t-(d_E-1)\Delta t)\}$. This implies that it is necessary to calculate to embedding parameters: time delay, Δt (the lag between data when reconstructing the state space), and embedding dimension, d_E (the dimension of the space required to unfold the dynamics). Although there have been numerous proposals the selection of the embedding dimension [14-15] and for the choice of time delay [16-17], they all are presented with the assumption of stationarity, which in our case does not hold.

Furthermore, in the context of nonstationarity, the notion of a "correct" embedding or delay is inappropriate as has been demonstrated in [18]. Instead it becomes important to remember that a sufficiently large embedding be chosen which will "contain" the relevant dynamics (as it may change from one dimensionality to another) as well as account the effects of noise, which tend to inflate dimension. In [19] the approach to "overembed" the time series to capture the dynamics as its dimension changes have been justified. Similar considerations govern the choice of the time delay. As the system changes from one dimension to another the effects of the time delay are changed. Thus a so-called "optimal" time delay in one embedding, becomes less so as the relevant dimension changes [20].

As the main interest in this work is one steep ahead prediction, we have considered that the optimal embedding parameters are those that produce a maximum gain and, hence, analysed our time series using this approach. However, one has to remember that these parameters would not be optimal when confronted with the same series for other years or when other function should be optimized.

<u>Divergence reconstruction</u>

As said before, state space reconstruction preserves certain information on the original system that originated the time series we are measuring. They are two types of preserved information: qualitative and quantitative.

- Qualitative information is that which allows a qualitative description of the dynamics, they are, for example singularity of the field, the closeness of an orbit, the stability of a fixed point, etc.
- Quantitative information can be of three different types, which involve metric, dynamical and topological invariants. Metric methods [21] depend on the computation of various fractal dimensions or scaling functions. Dynamical methods [22] rely on the estimation of local and global Lyapunov exponents and Lyapunov dimensions as well as on entropy. Topological methods [23] involve determination of specific topological invariants of the attractor as relative rotation rates for the unstable periodic orbits embedded in the attractor, etc.

However, all this information applies to the asymptotic behaviour of the system. By asymptotic behaviour, we mean the properties that prevail when time t is sufficiently large, $t \to \infty$. In our case as financial time series are transient, we need a local measure, not a global one, that reflects the actual status of the system. In this sense, we have been using the divergence of a dynamical system for the characterization and analysis of chemical transient reactors [24-26]. The divergence of the flow, which is locally equivalent to the trace of the Jacobian, measures the rate of change of an infinitesimal state space volume $V(t)$ following an orbit $\mathbf{x}(t)$. That means if $div[\boldsymbol{F}(\boldsymbol{x})] < 0 \ \forall \boldsymbol{x}$ in the state space then the flow of trajectories is volume-contracting, if $div[\boldsymbol{F}(\boldsymbol{x})] > 0 \ \forall \boldsymbol{x}$ the flow is volume-expanding and if $div[\boldsymbol{F}(\boldsymbol{x})] = 0 \ \forall \boldsymbol{x}$ then the flow is volume-preserving, i.e. for conservative systems. Furthermore, Liouville's theorem [27] states that:

$$V(t) = V(0) \cdot \exp\left[\int_0^t div\{\mathbf{F}[\mathbf{x}(\tau)]\} d\tau \right] \tag{1}$$

where

$$div\{\mathbf{F}[\mathbf{x}(t)]\} = \frac{\partial F_1[\mathbf{x}(t)]}{\partial x_1} + \frac{\partial F_2[\mathbf{x}(t)]}{\partial x_2} + \ldots + \frac{\partial F_d[\mathbf{x}(t)]}{\partial x_d} \tag{2}$$

Assuming we have a set of nearby trajectories in state space and using Eq. (1), it is possible to write:

$$V(t+h) = V(t) \cdot \exp\left[\int_t^{t+h} div[J(x)]d\tau\right] \tag{3}$$

expanding the exponential function in Taylor series, we obtain:

$$V(t+h) = V(t)\left[1 + \int_t^{t+h} div[J(x)]d\tau\right] \tag{4}$$

the integral term may be expressed, using the trapezium rule, as:

$$\int_t^{t+h} div[J(x)]d\tau = \frac{(div[J_{t+h}] + div[J_t])h}{2} \tag{5}$$

Inserting Eq. (5) into Eq. (4) and regrouping the terms we obtain:

$$\frac{(div[J_{t+h}] + div[J_t])}{2} = \frac{1}{h}\frac{V(t+h) - V(t)}{V(t)} \tag{6}$$

Hence, when $h \to 0$

$$div[J(x)] = \frac{\dot{V}(t)}{V(t)} \tag{7}$$

Furthermore, the divergence is preserved under state space reconstruction [27] and therefore it will reflect the local properties of our underlying dynamical system.

State space volume at time *t* may be calculated, assuming that the time step from one point to another in the time series is short enough that the Jacobian of the system has not substantially changed, using the determinant between close points in state space as:

$$V(t) = \det\begin{bmatrix} s(t) - s(t-\Delta t) & 0 & \dots & 0 \\ 0 & s(t-\Delta t) - s(t-2\Delta t) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & s(t-(d_E-1)\Delta t) - s(t-d_E\Delta t) \end{bmatrix} \tag{8}$$

and $\Delta V(t)$ is given by the difference between $V(t)$-$V(t-\Delta t)$.

Due to volume contraction in state space, characteristic of dissipative systems, V(t) could rapidly tend to zero and produce numerical problems when introduced as denominator in Eq. (7), for this reason we have used separately $V(t)$ and $\Delta V(t)$ avoiding the need to divide the two small numbers.

2.2. Trading strategies

A two step approach has been followed in applying the trading strategy. Even though it is not realistic, we assume in a first step that we can exchange our assets at no cost. Therefore, the number of transaction is not important. Furthermore, we have only tested one-step ahead prediction, i.e. $t+1$, based on all available information at time t. In the first case, we apply the following simple rule: if the variation of state space volume decreases, i.e. $\Delta V(t)>\Delta V(t-1)$, we change all our assets into $currency_2$ at $t+1$, on the contrary, we exchange all our assets into $currency_1$. The application of this strategy is equivalent to detect if the volume has a positive acceleration. We will then consider this acceleration as an index of the strength of the stock exchange. The net “profit”, also called return, is evaluated using the gain-loss function g [5] as:

$$g = \frac{y(t+1) - y(t)}{y(t)} \tag{9}$$

This function represents the rate of gain or loss incurred in one time step. The total gain-loss is calculated for all the time series as:

$$G = \sum_{i=d_E \cdot \Delta t}^{n} g_i \tag{10}$$

Therefore, in the first strategy if $\Delta V(t)>\Delta V(t-1)$, we will change all our assets into $currency_2$ at $t+1$, if we are confronted for the first time to a decrease in ΔV, if not then no action is performed. As we will see later on, this strategy for real financial time series produces a considerable amount of transactions since our ΔV is oscillating around zero. In case of transaction costs this strategy would fail.

To reduce the number of transactions and consider only the most relevant, we have introduced a second trading criterion based on the same concepts, i.e. if $\Delta V(t)>\Delta V(t-1)$, we will change all our assets into $currency_2$ at $t+1$, but with a limit for the minimum state space volume that will depend on the embedding dimension at which we are working. Only when $|V(t)| > \lim$ then the former criterion is checked. This approach reduces the number of transactions and, therefore, we have applied it, for the case of analysing the results when transaction costs are involved, a fixed 0.2% cost for each transaction which in practice means that we multiply by 0.998 our assets after each change.

2.3. A simple case example

In order to understand the proposed approach, let us consider a simple case of an exchange currency in the form of a normalised sinusoidal function, y = [sin(x)+1.1]/2.1. Figure 1a represents the function, whereas in fig.1b and 1c the first derivative and the state space volume with its sign are represented, respectively. In the case of a one-dimensional system both values are identical. Figure 1d represents the variation of state space volume that in this case would represent the second derivative of the system.

According with the trading strategy defined previously, when the change in the state space volume decreases, we will move, in the next step $t+1$, our assets to $currency_2$ whereas when the change in state space volume increases we will change our assets – at $t+1$- into $currency_1$. This can be seen in fig. 1d represented by red and green colours. Following this strategy and starting with 100 units of account in $currency_1$, Fig. 2 represents evolution of the amount of $currency_1$ *and* $currency_2$ during the time. The final values, in this simple example, are 2090.1 or 4001.8 if we consider $currency_1$ or $currency_2$, respectively. As can be seen, even though we have applied the first trading strategy, in this case the number of transactions is limited due to the smooth nature of the function.

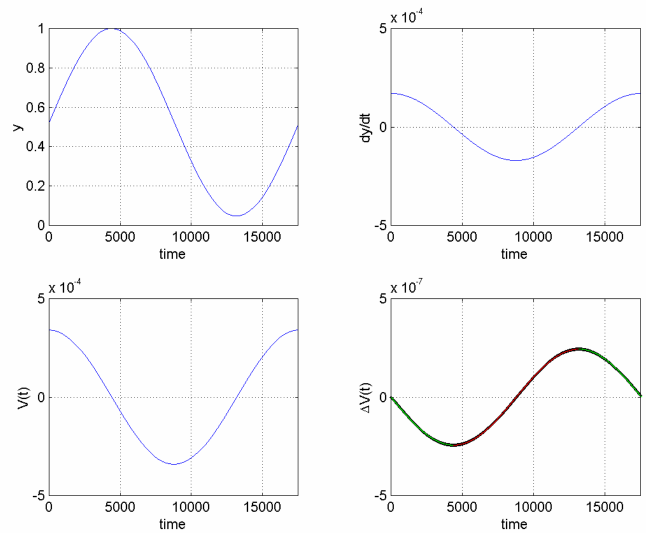

Figure 1. a/ Sinusoidal function; b/ first derivative; c/ State space volume; d/ State space volume change (green funds in $currency_2$; red funds in $currency_1$).Reconstruction parameters: Δt=2, d_E=1.

Let us assume that we do not know *a priori* the optimum values for the embedding parameters. In this case we can analyse how the net profit function changes as a function of the time delay and embedding dimension, see Figure 3. In this case the optimum values are Δt = 2 and d_E = 1. As can be seen at low embedding dimensions and time delays we are able to predict correctly the behaviour of the time series. However, as we start to increase the dimension and the time delay our prediction capabilities start to fail and our gain-loss function became negative.

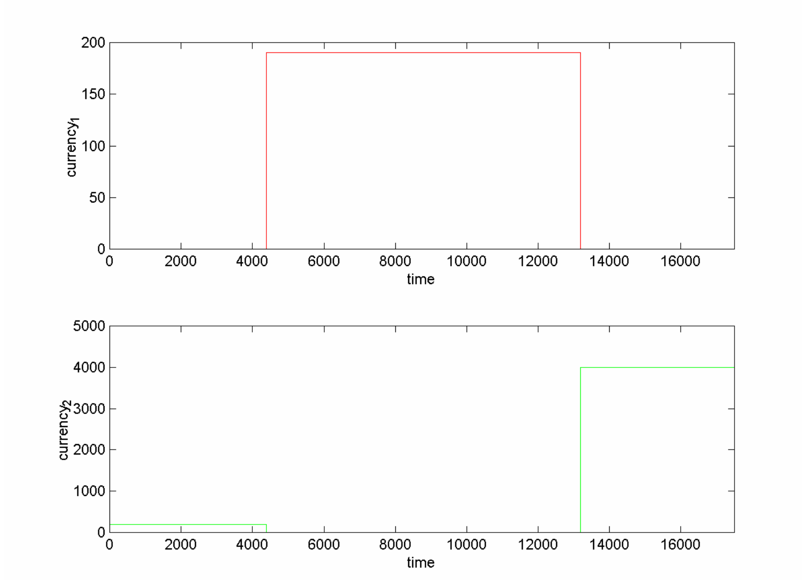

Figure 2. Exchanges between *currency₁* and *currency₂* for the sinusoidal function following the trading strategy defined in Section 2.2 and starting with 100 units in *currency₁*.

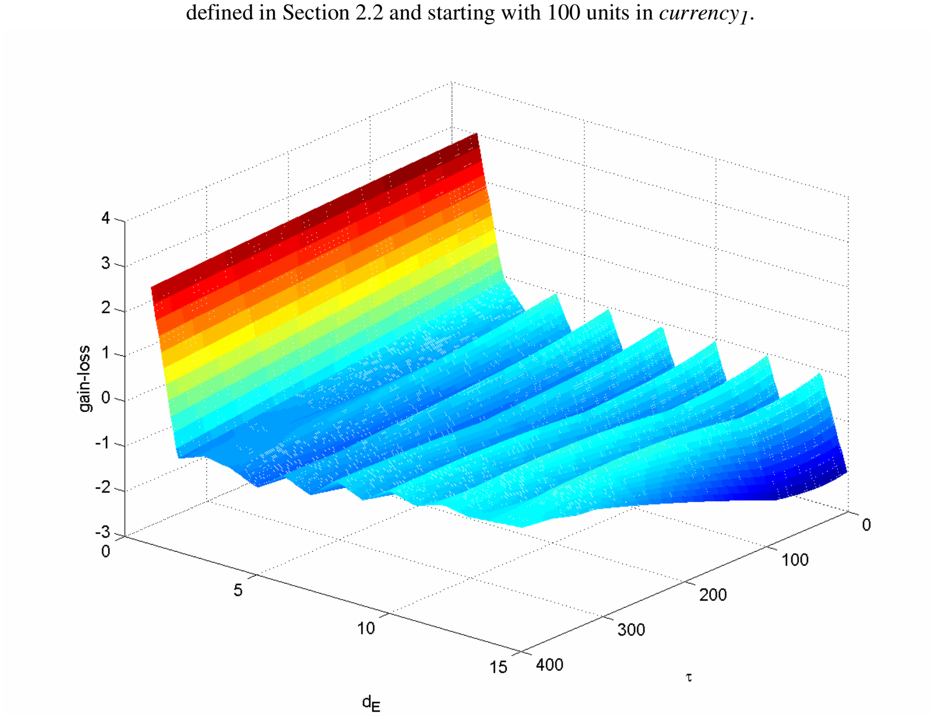

Figure 3. Gain-loss function for the considered time delays (between 2 and 400) and embedded dimensions (between 1 and 15) for the sinusoidal function.

3. Analysis and results

In this work, we have applied two trading strategies, with and without transaction costs, to high-frequency currency exchange data from the HFDF96 data set provided by Olsen & Associates. The time series studied are the exchange rates between the US Dollar and 18 other foreign currencies from the Euro zone; i.e. Belgium Franc (BEF), Finnish Markka (FIM), German Mark (DEM), Spanish peseta (ESP), French Frank (FRF), Italian Lira (ITL), Dutch Guilder (NLG), and finally ECU (XEU); and from outside the Euro zone: Australian Dollar (AUD), Canadian Dollar (CAD), Swiss Frank (CHF), Danish Krone (DKK), British Pound (GBP), Malaysian Ringgit (MYR), Japanese Yen (JPY), Swedish Krona (SEK), Singapore Dollar (SGD), and South African Rand (ZAR). We have used only the bid prices and we have not performed any transformation to the original data sets.

3.1. Trading without transaction costs involved

As a first approach we have put no limitations to the number of transactions and consider that when $\Delta V(t)$ decreases, we change our assets into $currency_2$ at the price at $t+1$.On the contrary, we move our assets to $currency_1$. In order to assess the level of predictability, we have tested the gain-loss function for values of time delay between 2 and 400 and embedding dimensions between 1 and 15. This values have been selected in agreement with our previous analysis using nonlinear time series methods for this high frequency data set.

Table 1 summarizes the results for each foreign currency. In the first column the percentage of values for which a positive value for the gain-loss function is obtained are represented. As can be seen a mean value of 66.5 of positive predictions is shown. This value in some foreign currency series is higher than 90%. Furthermore, the optimum time delay and embedding dimension are represented, as well as the optimal gain-loss function that oscillated between 0.08 and 0.41. This implies a gain in percentage higher than 8 and 41%. Figures 4-7 show for the Australian Dollar (AUD) and the Belgium Franc (BEF) the gain-loss function for each time delay and embedding dimension tested as well as its histogram. This analysis is reminiscent of a similar approach developed in ref. [28] using RQA analysis to derive embeddings and delays. As can be seen there is an asymmetry of the distribution. Since all the probability distribution functions are towards the right side –exception the Italian Lira, the Malaysian Ringgit and the Singapore Dollar, then our probability for obtaining a positive gain by selecting randomly one time delay and embedding dimension is higher than obtaining a negative gain. A similar result, but with lower g values, max. 0.02, was obtained in [5] using a simple rule, antipersistance, for the dollar-yen exchange time series.

Table 1. Best parameters and predictability results without transaction costs for the currency exchange time series considered.

Currency	Δt^{opt}	d_E^{opt}	$\sum g^{opt}$	*%gain*
AUD	385	1	0.21	75.9
BEF	184	6	0.27	65.8
CAD	276	6	0.10	51.6
CHF	67	3	0.28	90.2
DEM	89	7	0.20	75.2
DKK	2	3	0.20	71.4
ESP	273	1	0.39	64.5
FIM	59	10	0.25	55.8
FRF	192	11	0.20	69.5
GPB	295	4	0.20	94.3
ITL	189	1	0.16	36.8
JPY	212	12	0.21	86.0
MYR	242	3	0.04	37.5
NLG	218	2	0.19	73.5
SEK	182	1	0.28	56.4
SGD	330	1	0.08	38.8
XEU	29	1	0.18	58.3
ZAR	191	7	0.37	95.5

Δt^{opt} and d_E^{opt} indicate, respectively, the optimum time delays and embedding dimensions for state space reconstruction in the sense of higher gain-loss function, i.e. $\sum g^{opt}$ *%gain* refers to the number of times in which there was a net gain for all combinations of reconstruction parameters, i.e. (Δt between 2-400 and d_E between 1-15).

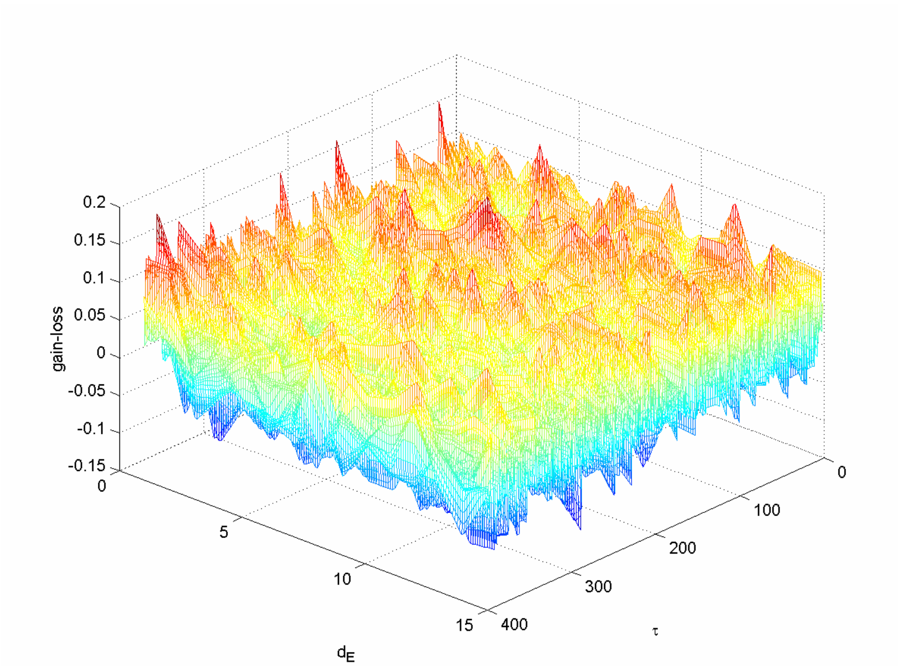

Figure 4. Gain-loss function for the considered time delays and embedded dimensions for the Australian Dollar (AUD).

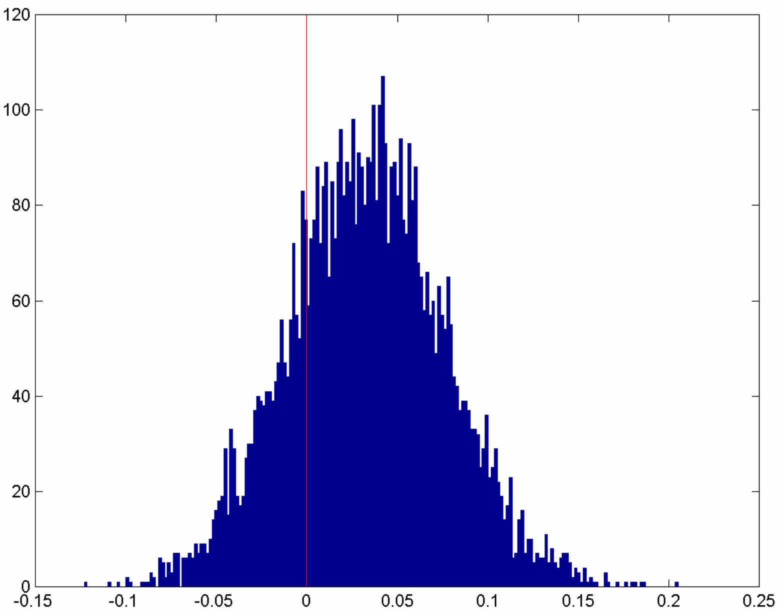

Figure 5. Histogram of the gain-loss function for the Australian Dollar (AUD). Time delay between 2 and 400; embedding dimension between 1 and 15.

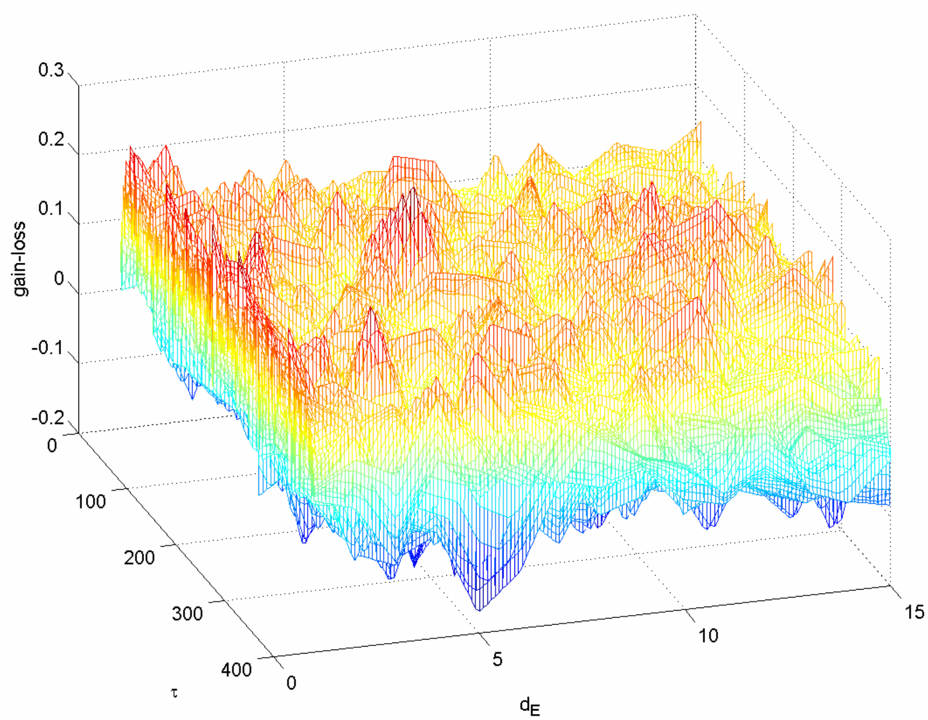

Figure 6. Gain-loss function for the considered time delays and embedded dimensions for the Belgium Franc (BEF).

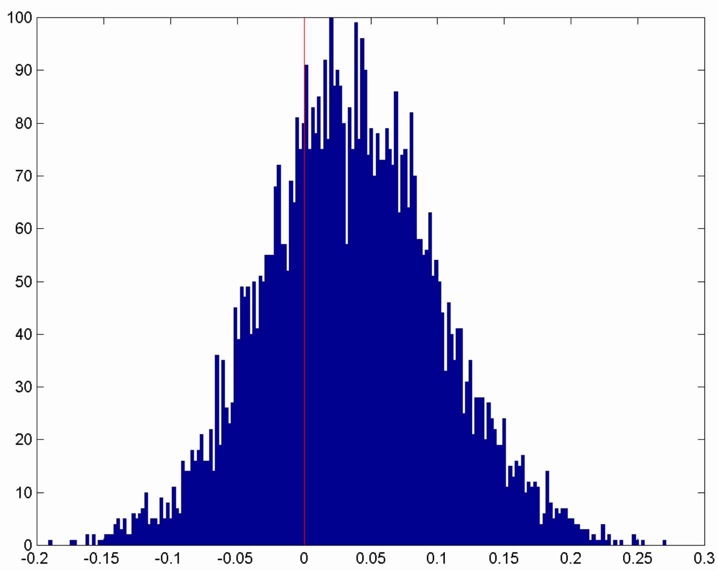

Figure 7. Histogram of the gain-loss function for the Belgium Franc (BEF). Time delay between 2 and 400; embedding dimension between 1 and 15.

Figures 8-11 show two examples corresponding to the Italian Lira and to the British Pound of the results obtained using the optimal reconstruction parameters. As can be seen there is a continuous gain all over the year even though both time series have a completely different behaviour from the point of view of currency exchange, i.e. one series is increasing the other decreasing.

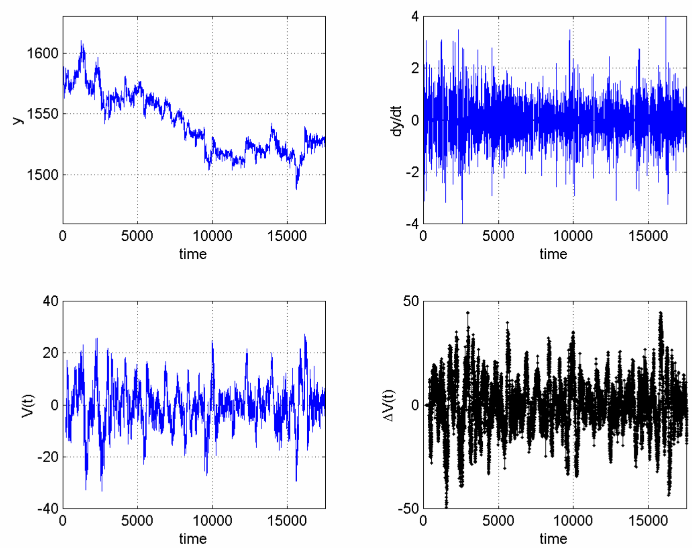

Figure 8. No transaction costs, no state space volume limitations: a/ Italian lira- US dollar time series (bid); b/ first derivative; c/ State space volume; d/ State space volume change. Reconstruction parameters: Δt=189, d_E=1.

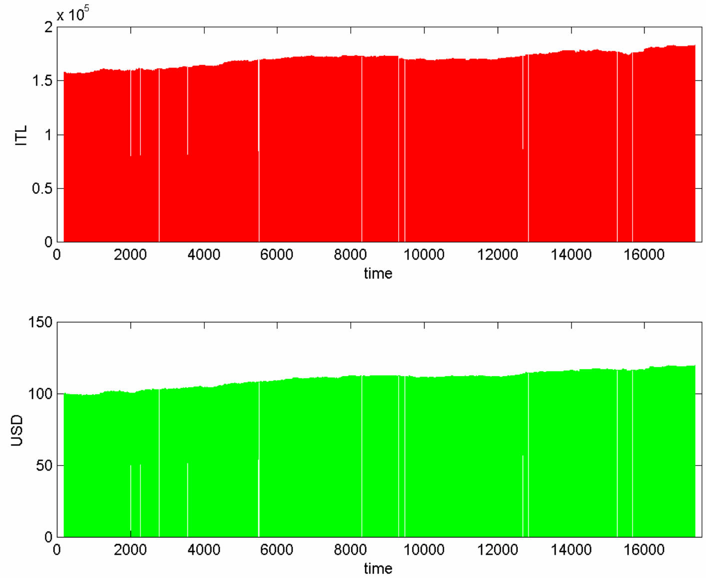

Figure 9. Exchanges between *ITL* and *USD* using no transaction costs and no state space volume limitations trading strategy and starting with 100 units in *USD*.

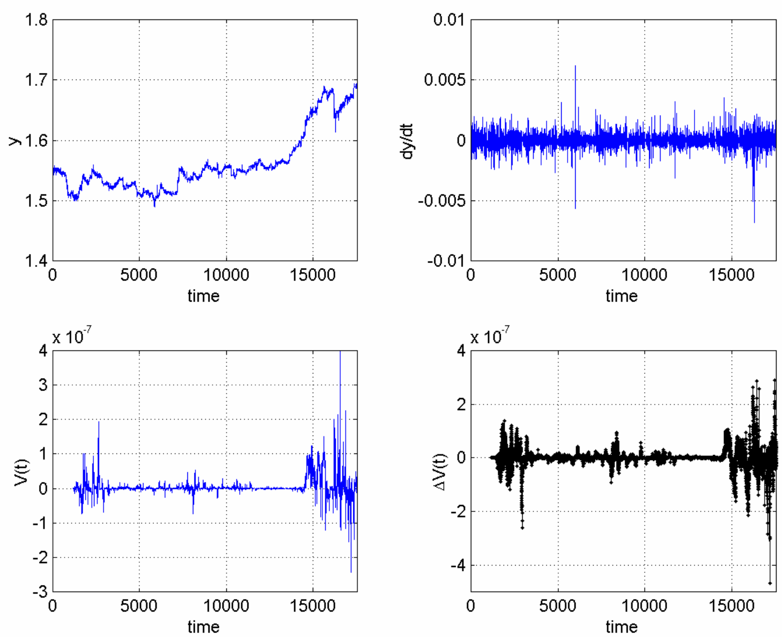

Figure 10. No transaction costs, no state space volume limitations: a/ US dollar – British Pound time series (bid); b/ first derivative; c/ State space volume; d/ State space volume change. Reconstruction parameters: Δt=295, d_E= 4.

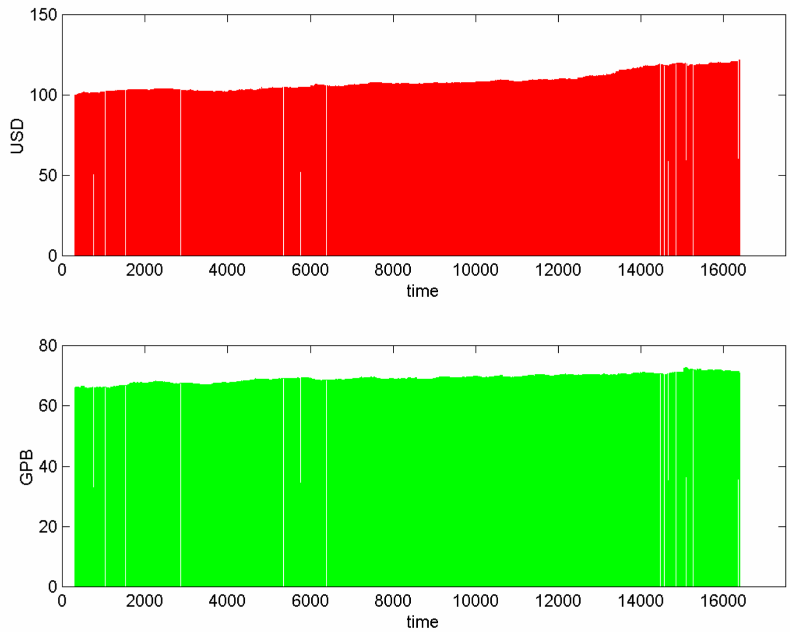

Figure 11. Exchanges between *USD* and *GPB* no transaction costs and no state space volume limitations trading strategy and starting with 100 units in *USD*.

Table 2. Best parameters and predictability results without transaction costs for the currency exchange time series considered by optimising the state space volume at which a transaction is allowed.

Currency	Δt^{opt}	d_E^{opt}	$\sum g^{opt}$	$\%gain$
AUD	385	1	0.22	83.9
BEF	133	1	0.34	99.8
CAD	2	1	0.11	73.5
CHF	47	1	0.31	97.4
DEM	2	1	0.25	96.6
DKK	4	1	0.23	99.5
ESP	13	1	0.51	99.9
FIM	2	1	0.32	99.4
FRF	3	1	0.27	99.9
GPB	4	1	0.20	96.4
ITL	42	1	0.20	93.7
JPY	37	1	0.23	100.0
MYR	2	1	0.08	83.4
NLG	36	1	0.22	97.7
SEK	2	1	0.30	99.7
SGD	3	1	0.10	72.8
XEU	2	1	0.22	84.0
ZAR	171	7	0.37	99.9

With this trading strategy, it is clear that there is no limitation in the number of transactions to perform, see Figs. 9 and 11. Therefore, once transactions costs are included it seems evident that it will be difficult to obtain a net gain. For this reason, we have tested a modified strategy in which we use the value of the state space volume to decide if a transaction should be performed or not. For this reason, we have run the same algorithm but changing at the same time the $|V|$ at which transactions are allowed. The results are summarised in Table 2. As can be seen, only for one currency, i.e. Australian dollar, we have obtained the same optimum value for state space reconstruction as in Table 1. Furthermore, one should notice that in this case the optimum values are found using an embedding dimension of one which in practical terms means we are calculating derivatives of the time series, the exceptions is the South African Rand – however for this time series in the second place a value close to the optimal is found with embedding dimension of one, i.e. ZAR(d_E=1, τ=6, $\sum g^{opt}$ =0.35)-. In this sense typical instruments of Technical Analysis [8-9] using by chartists are justified. Another striking feature is that with this strategy, the number of combinations of time delay and embedding dimension at which we will obtain a net gain, using the optimal value of $|V|$ at which start a transaction, is quite high. In other words, the probability that for each time delay and embedding dimension we choose, there is a certain value of $|V|$ for which we could obtain a net gain has an average value of 93%. These two facts, i.e. optimal embedding dimension equal to one and high percentage of gain explain why, despite all the work on Efficient Market Hypothesis, there has been a considerable amount of traders that use Technical Analysis tools [8-9] to trade in financial markets.

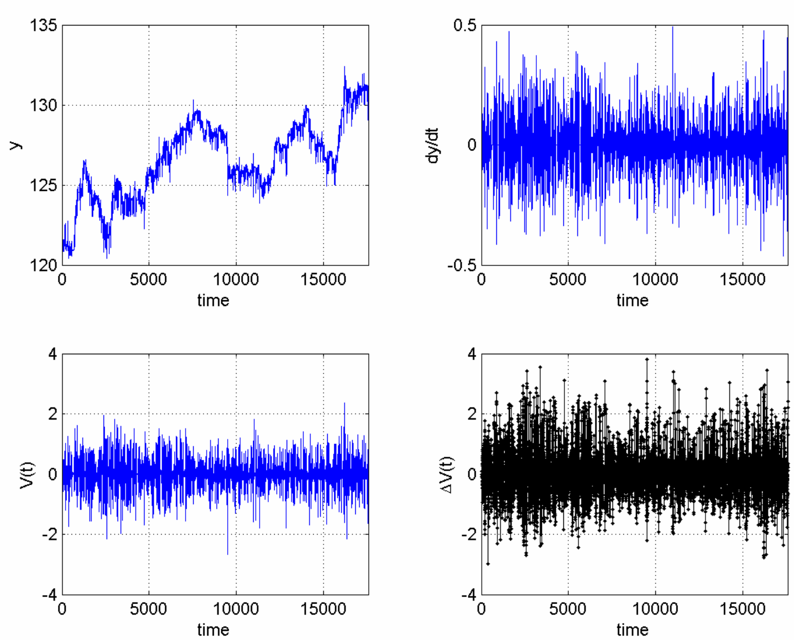

Figure 12. No transaction costs, state space volume limitation: a/ Spanish Peseta-US dollar time series (bid); b/ first derivative; c/ State space volume; d/ State space volume change. Reconstruction parameters: Δt=13, d_E = 1; and with $|V|>lim$ (0.51) as minimum state space volume to trade.

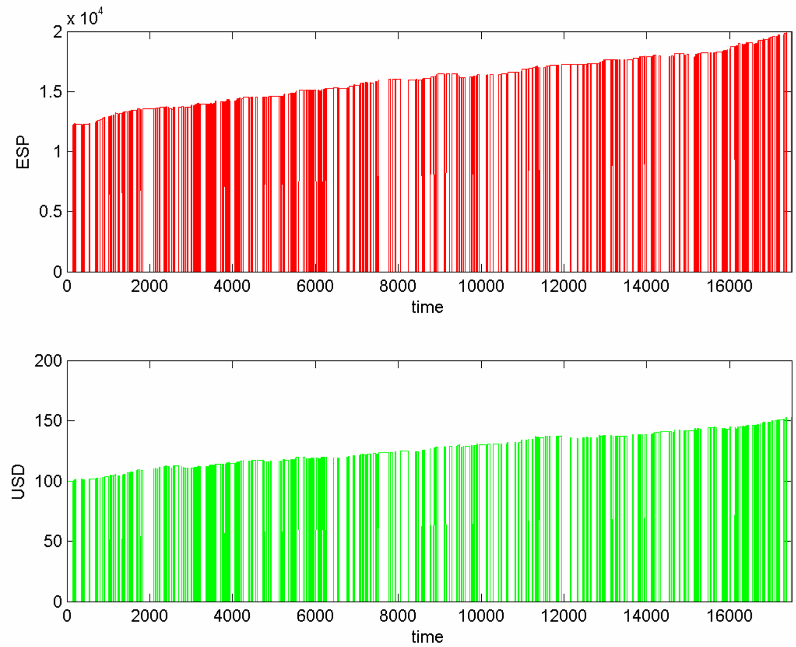

Figure 13. Exchanges between *USD* and *ESP* with no transaction costs and state space volume limitation, i.e. |*V*|>*lim* (0.51) and starting with 100 units in *USD*.

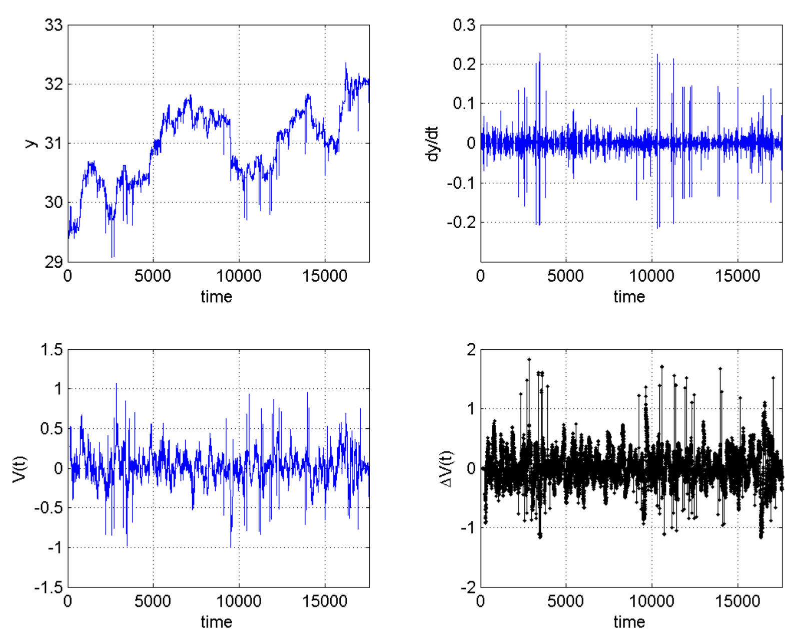

Figure 14. No transaction costs, state space volume limitation: a/ Belgium Franc-US dollar time series (bid); b/ first derivative; c/ State space volume; d/ State space volume change. Reconstruction parameters: Δt=133, d_E = 1; and with |*V*|>*lim* (0.16) as minimum state space volume to trade.

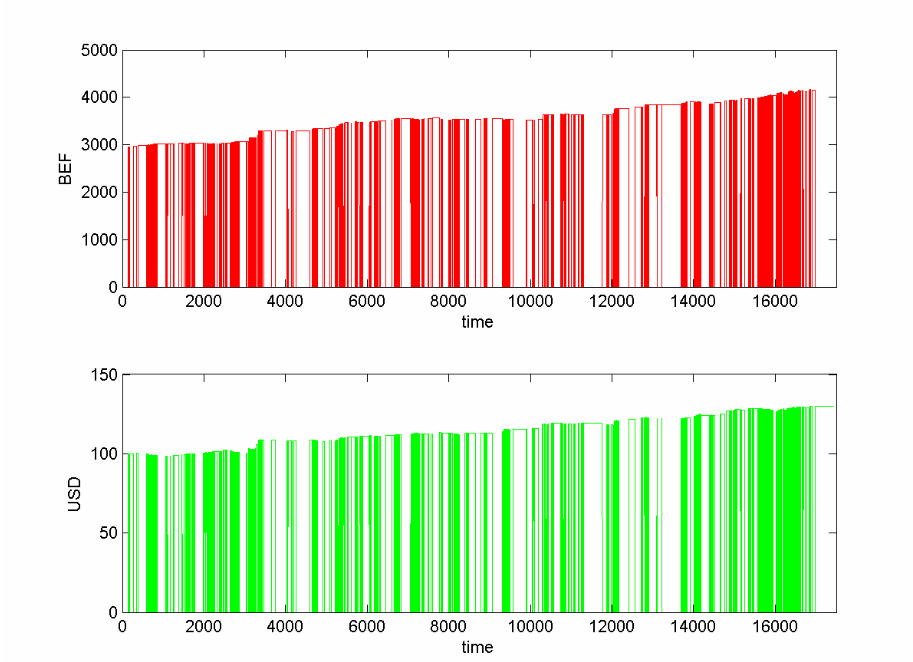

Figure 15. Exchanges between *USD* and *BEF* with no transaction costs and state space volume limitation, i.e. $|V|>lim$ (0.16) and starting with 100 units in *USD*.

As can be seen in the two selected examples - figs. 12-15- , i.e. Spanish peseta and Belgium Franc, the number of transactions has decreased when compared with the first trading methodology but it is probably still too high to be able to deal with transaction costs. In any case this approach allow to reduce the number of transactions according to their importance in terms of state space volume values.

3.2. Analysis considering transaction costs

For this case we have considered a fixed amount of transactions costs equal to 0.2% for each transaction [29]. It is clear that in this case we need to optimize the number of transactions. In this sense we have selected to use an absolute value of the state space volume, as explained above, that reflects in a certain sense the importance of the change in the time series. Therefore, if $|V|>limit$ we apply the same methodology that in the case without transaction costs. Table 3 summarises the results. As can be seen, in this case the gain decreases but still fluctuates around 11%. Furthermore, the number of combinations of time delay and embedding dimension for which it is possible to obtain a net gain also decreases considerably when compared with the results in Tables 1-2. Another interesting feature is that in the case of a 0.2% of transaction costs, only six optimal strategies are found for embedding dimension of one –which does not imply that net gain may be still obtained-. Increasing the transaction costs up to 1% then no optimal strategy is found (results not shown) with embedding dimension of one.

Table 3. Best parameters and predictability with 0.2% transaction costs, at each transaction, for the currency exchange time series considered optimising the state space volume at which a transaction is allowed.

Currency	Δt^{opt}	d_E^{opt}	$\sum g^{opt}$	*%gain*
AUD	4	1	0.11	66.0
BEF	56	1	0.15	89.1
CAD	125	4	0.04	36.2
CHF	14	4	0.19	96.1
DEM	7	4	0.12	78.2
DKK	11	3	0.10	86.0
ESP	11	1	0.14	93.2
FIM	8	1	0.12	78.5
FRF	10	1	0.09	79.7
GPB	10	2	0.14	94.3
ITL	52	1	0.05	28.7
JPY	335	14	0.13	99.6
MYR	311	5	0.02	50.5
NLG	392	5	0.11	83.1
SEK	3	5	0.10	69.0
SGD	125	1	0.02	7.0
XEU	25	2	0.09	57.2
ZAR	129	2	0.26	99.4

Figures 16-19 show the results for two examples. As can be seen in these cases the net gain obtained assuming 0.2% of transaction costs each time we move our assets from one currency to another, decreases. Therefore the number of transactions is reduced considerably. Of course, depending of these costs is always possible, following this trade strategy, to find an optimal set of parameters.

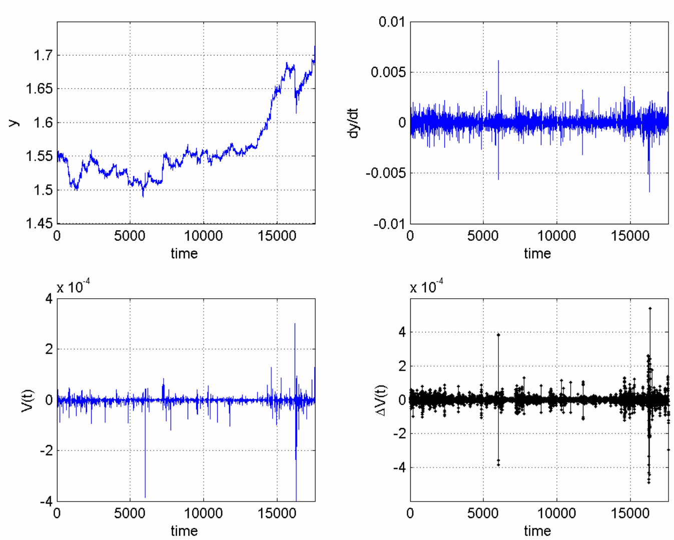

Figure 16. Fixed transaction costs (0.2%) and optimum state space volume: a/ US dollar-British Pound time series (bid); b/ first derivative; c/ State space volume; d/ State space volume change. Reconstruction parameters: $\Delta t = 10$, $d_E = 2$, $|V|>1.3\cdot10^{-4}$).

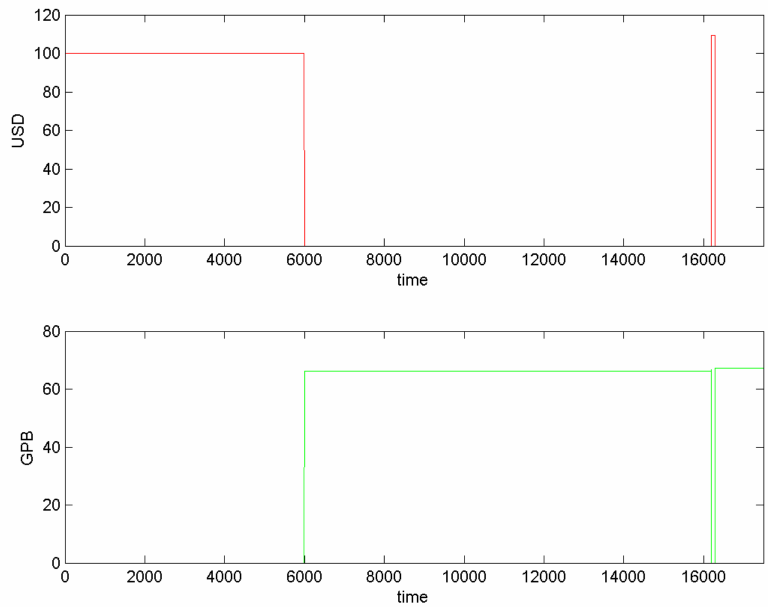

Figure 17. Exchanges between GPB and USD with fixed transaction costs (0.2%) and state space volume limitation, starting with 100 units in USD.

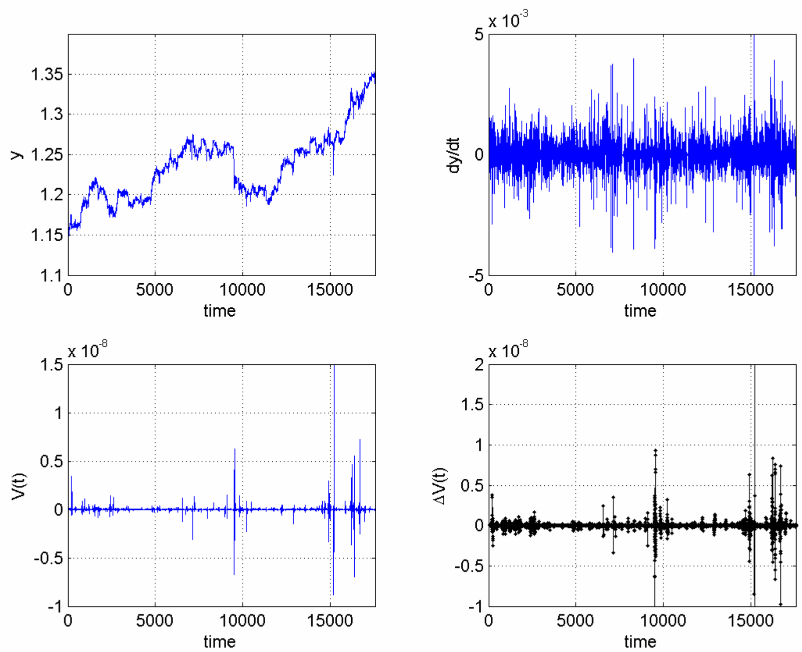

Fig. 18. Fixed transaction costs (0.2%) and optimum state space volume: a/ US dollar-Swiss Frank time series (bid); b/ first derivative; c/ State space volume; d/ State space volume change. Reconstruction parameters: $\Delta t=14$, $d_E=4$, $|V|>1.5\cdot 10^{-9}$).

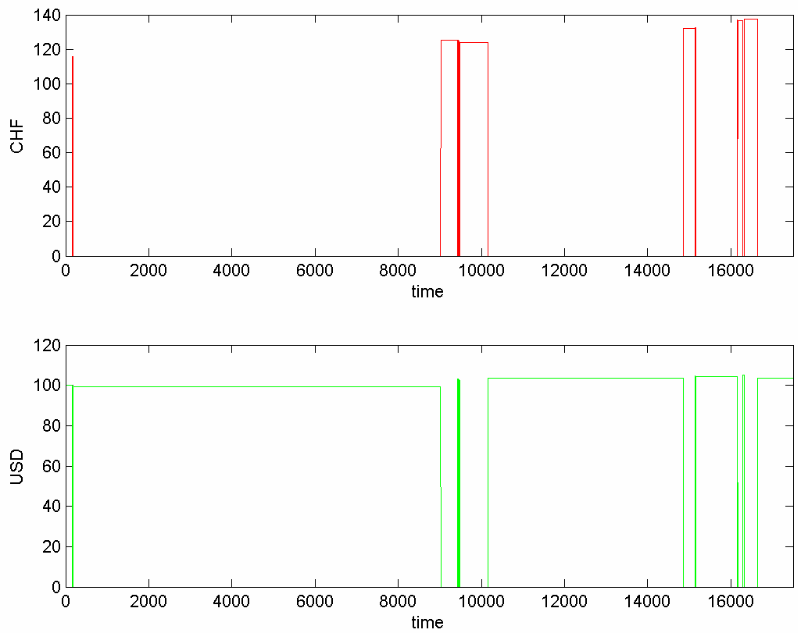

Figure 19. Exchanges between CHF and USD with fixed transaction costs (0.2%) and state space volume limitation, starting with 100 units in USD

3.3. Comparison with Relative Strength Index (RSI)

Over the years, investors have developed many different indicators which attempt to measure the velocity or the acceleration of price movements and are used to determine a trading strategy. These indicators are grouped together under the heading of momentum. Some of the more popular indicators are: Rate of Change (ROC), Relative-Strength Index (RSI), moving average convergence-divergence (MACD), and stochastic oscillator [8-9].

In order to compare with a well-known trading strategy, we have chosen the relative strength index (RSI) [9]. This index is a popular indicator created by US analyst J. Welles Wilder jr. and measures the ratio of the sum of the up-moves to down-moves normalising the calculation between 0 and 100. We have taken the complete time series and optimize the trading values by modifying the size of the band (outside of which we will change) and window (past values used to calculate the average of up-moves and the down-moves). Table 4 summarises the results. As can be seen, the optimised RSI gives slight greater gains than the state space in fifteen out of eighteen currencies.

Table 4. Best parameters without transaction costs for the currency exchange time series considered using the RSI momentum.

Currency	b^{opt}	w^{opt}	$\sum g^{opt}$
AUD	17	5	0.28
BEF	8	10	0.42
CAD	3	30	0.14
CHF	1	24	0.32
DEM	1	68	0.27
DKK	2	10	0.25
ESP	1	14	0.47
FIM	1	30	0.33
FRF	1	5	0.31
GPB	1	12	0.26
ITL	2	25	0.30
JPY	1	28	0.28
MYR	6	24	0.08
NLG	3	58	0.19
SEK	4	29	0.36
SGD	2	9	0.11
XEU	3	10	0.32
ZAR	10	24	0.39

We have also compared with RSI for the case of fixed transaction costs 0.2%, see Table 5. Contrary to the case previous case, the state space volume method outperforms the traditional trading strategy by more than one order of magnitude. In this case, we can conclude that our trading strategy produces better results that traditional techniques usually employed by chartists.

3.4. Opposite criterion

Another critic to a considerable number of trading techniques is that if, instead of using the selected trading criterion, we compute the opposite, then we will obtain similar results. Table 5 summarises the results applying the opposite criterion, i.e. if $\Delta V(t)<\Delta V(t-1)$, we will change all our assets into $currency_2$ at $t+1$, whereas in Table 6 the RSI criterion is also applied following the opposite criterion, i.e. we change outside of the defined band. As can be seen from the tables, the opposite strategy in both cases will produce losses.

Table 5. Best parameters and gain with 0.2% fixed transaction costs for the currency exchange time series considered using the RSI momentum.

Currency	b^{opt}	w^{opt}	$\sum g^{opt}$
AUD	27	104	0.013
BEF	7	359	0.018
CAD	20	88	-0.024
CHF	14	240	0.005
DEM	23	88	0.002
DKK	10	342	0.006
ESP	12	189	0.002
FIM	9	274	0.004
FRF	12	206	0.009
GPB	11	223	0.003
ITL	5	444	0.002
JPY	25	78	0.003
MYR	26	108	0.000
NLG	20	121	0.007
SEK	17	98	-0.007
SGD	32	98	0.000
XEU	10	291	0.005
ZAR	32	108	-0.004

Table 6. Results without transaction costs for the currency exchange time series considered using the opposite state space volume strategy, i.e. if $\Delta V(t)<\Delta V(t-1)$, we will change all our assets into $currency_2$ at $t+1$ (Optimum parameters from Table 1) and the opposite RSI strategy.

Currency	$\sum g^{\Delta V_{crit}}$	$\sum g^{RSI}$
AUD	-0.07	-0.16
BEF	-0.21	-0.14
CAD	-0.09	-0.12
CHF	-0.12	-0.10
DEM	-0.13	-0.15
DKK	-0.14	-0.11
ESP	-0.31	-0.28
FIM	-0.15	-0.10
FRF	-0.15	-0.19
GPB	-0.07	-0.16
ITL	-0.17	-0.31
JPY	-0.12	-0.11
MYR	-0.05	-0.06
NLG	-0.11	-0.10
SEK	-0.23	-0.27
SGD	-0.09	-0.12
XEU	-0.16	-0.21
ZAR	-0.10	-0.11

4. Conclusions

In this work a new trading methodology based on state space volume calculation has been introduced. This methodology has been tested using eighteen high-frequency foreign exchange time series with and without transaction costs. The results are in apparent contradiction with the Efficient Market Hypothesis (EMH) which states that no profitable information about future movements can be obtained by studying the past prices series. In our analysis an optimum mean value of approximately 25% gain may be obtained in those series without transaction costs and an optimum mean value of approximately 11% gain assuming 0.2% of costs in each transaction. The trading strategy has been compared with the RSI (Relative-Strength Indicator) used for trading in financial market. Even though slight better results, in terms of net gain, are obtained when transaction costs are not considered, when a fixed 0.2% transaction cost is introduced, our state space volume algorithm outperforms RSI by more than one order of magnitude.

Efficient Market Hypothesis considers that financial markets are impossible to forecast. In this work, we have demonstrated that it is possible, even after considering trading costs, to obtain a net gain. However, to dismiss the EMH the forecasting should be done in real-time since in real markets investors' current and future forecast of payoffs affect their current and future trades which in turns affect returns, i.e. there is a feedback. Furthermore, the analysis presented in this work is not completely blind in the sense that we only use past information. This is due to the fact that we have used the complete time series to obtain optimal values of reconstruction parameters for showing that there are values for which a net gain is possible. However, the high percentage of net gain cases indicates that is not difficult to find an adequate combination and update iteratively on real-time as data become available. This may be done using a similar approach as the RSI, which here we have also analysed using all the information available.

If investors start to apply this forecasting methodology the temporary forecastability that exists will quickly disappear and, hence, the EMH will hold. In this sense, by applying more sophisticated trading strategies the financial markets will become more efficient.

Finally, we may conclude that in terms of prediction power, high-frequency foreign exchange time series have a different behaviour from a random walk (see Appendix 1), i.e. are more predictable. In this sense we may say that a certain amount of determinism is embedded in the analysed financial time series that made their prediction more accurate that a random walk.

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Appendix 1. Random walks and high frequency foreign currency echange financial time series

The objective of this appendix is to compare the analysed high-frequency currency exchange financial time series with random walks. For these reasons we have used a different approach than in the main report. In this case, we have normalised our time series following the same approach as discussed in [15], i.e. we consider the logarithmic middle price y_m, which can be calculated as follows:

$$y_m = \frac{\log(p_{bid}) + \log(p_{ask})}{2} \tag{A1}$$

where p_{bid} and p_{ask} are the bid and ask prices of the US Dollar with respect to some currency, respectively. In order to compare the different data sets analysed, we have normalised data sets between δ and $1+\delta$ and obtained a normalised logarithmic middle price y as follows

$$y = \frac{ym - y_m^{\min}}{y_m^{\max} - y_m^{\min}} + \delta \tag{A2}$$

The δ value ($1.0 \cdot 10^{-3}$) is necessary to avoid division by zero when changing from one currency to the another. For the case of random walk we have generated the time series using the random number generation utility of MATLAB with the same number of points (17568, one point each half hour for 366 days) as the financial time series and we have afterwards normalised them in the same way.

A1. Probability distribution functions

Carrying out a similar analysis as in [4-5], it is possible to see, Fig. 1A, that the probability distribution functions found in our high-frequency foreign exchange financial time series exhibits a fat – tailed distribution which is in disagreement with the Gaussian distribution (blue circles) and in agreement with previous studies in Econophysics [4-5] (for a detailed discussion on the implications of these distribution functions the reader is referred to above mentioned references).

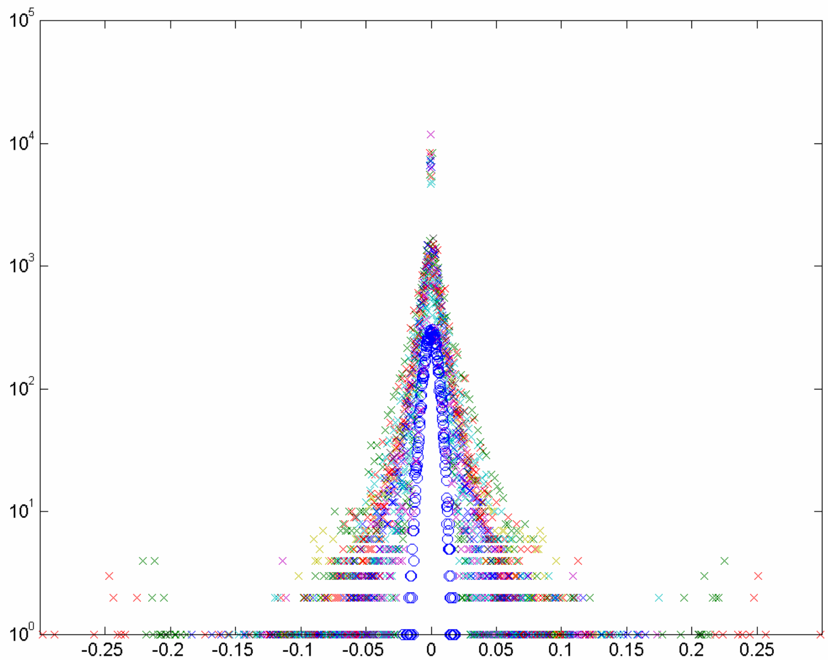

Figure 1A. Probability distribution functions of the first difference, i.e. $y(t+1)$-$y(t)$, for the eighteen normalised high-frequency foreign exchange time series compared with a normalised random walk (Gaussian distribution) time series, blue circles.

A2. Comparison on prediction

To compare high-frequency foreign currency exchange time series with random walks in terms of forecasting power, we have generated and normalised 20 random walks time series. The results are summarised in Table A1.

The same approach was applied to the normalised high-frequency foreign exchange time series. The results are shown in Table A2. As can be seen there is a difference between the normalised time series (considering bid and ask) and the non-normalised time series (using only bid) in Table 1. The most striking feature is the increase in the net gain. This is mainly due to the fact that the series is stretched to occupy the whole range (between 0.001 and 1.001). Furthermore, there is also a small decrease in the number of times in which a net gain is obtained when analysing all the combination of reconstruction parameters.

Table A1. Best parameters and predictability results without transaction costs for the random walk time series considered.

Currency	%gain	τ^{opt}	d_E^{opt}	$\sum g^{opt}$
RAND1	56.7	38	15	14.7
RAND2	22.9	351	14	16.5
RAND3	55.9	153	13	17.4
RAND4	73.1	127	6	13.9
RAND5	47.9	27	11	18.3
RAND6	6.8	126	15	6.7
RAND7	50.8	110	13	15.7
RAND8	53.8	161	3	19.2
RAND9	51.7	42	4	16.7
RAND10	39.9	118	2	24.4
RAND11	25.4	245	15	13.2
RAND12	55.8	271	13	17.0
RAND13	52.9	85	9	23.8
RAND14	41.2	353	9	16.1
RAND15	61.7	28	12	18.6
RAND16	31.6	176	11	8.8
RAND17	77.7	65	8	15.7
RAND18	7.7	186	14	7.8
RAND19	40.0	73	10	23.8
RAND20	27.8	59	8	17.8

Table A2. Best parameters and predictability results without transaction costs for the currency exchange time series considered.

Currency	%gain	τ^{opt}	d_E^{opt}	$\sum g^{opt}$
AUD	74.4	13	5	18.4
BEF	61.4	134	1	279.7
CAD	50.3	31	7	25.6
CHF	84.2	4	6	16.5
DEM	71.4	36	1	27.3
DKK	84.2	3	1	72.2
ESP	65.1	39	5	74.7
FIM	61.6	4	4	26.8
FRF	68.1	18	1	23.4
GPB	66.3	142	6	14.9
ITL	41.6	176	11	41.5
JPY	74.2	6	2	28.8
MYR	48.7	188	14	26.1
NLG	70.6	3	6	98.1
SEK	53.4	3	13	83.9
SGD	47.0	115	1	667.0
XEU	56.3	114	1	36.3
ZAR	95.6	28	2	17.3

To discriminate between the two time series sets, we have defined as null hypothesis that the median - less dependent on extreme values and more appropriate for skewed distributions- of our financial time series is the median of a random walk time series for the optimal prediction obtained using the best combination of reconstruction parameters, i.e. time delay and embedding dimension. We have applied the non-parametric sign (or median) test [30] to accept or reject such a null hypothesis to *%gain* and $\sum g^{opt}$ values. The sign test states that the hypothesis to have the same median is rejected at 5% level of significance if $\left| n_{median} / n - 1/2 \right| > 1/\sqrt{n}$, where n_{median} refers to the number of observation lower than the median of the random walk time series and n is the total number of observations. If we apply the test to the *%gain,* we obtain a value for the lhs of 0.33 whereas applying it to $\sum g^{opt}$ we obtain 0.39, which are both bigger than 0.24, rhs of the inequality.

We may conclude that, in terms of prediction power, high-frequency foreign exchange time series have a different behaviour from a random walk, i.e. are more predictable. In this sense we may say that a certain amount of determinism is embedded in the analysed financial time series that made their prediction more accurate that a random walk.